Isolated boundary singularities of positive solutions to quasilinear elliptic equations

P. T. Nguyen
Technion, Haifa, Israel
Joint work with L. Véron

Abstract We study nonnegative solutions of the quasilinear equation

$$-Δu + |∇u|^q = 0 \quad \text{in } Ω$$

where $1 ≤ q ≤ 2$ and $Ω$ is a $C^2$ bounded domain in $\mathbb{R}^N$ with $∂Ω$ containing the origin $0$. We show that

i) When $q = 1$, for any $k > 0$, there exists a unique solution $u_{k,0}$ of (0.1) with boundary data $kδ_0$ where $δ_0$ is the Dirac mass at $0$. Moreover, $\lim_{k→∞} u_{k,0} = ∞$.

ii) When $1 < q < q_c = \frac{N+1}{N}$, for any $k > 0$, there exists a unique solution $u_{k,0}$ of (0.1) with boundary data $kδ_0$. Moreover $u_{∞,0} = \lim_{k→∞} u_{k,0}$ is a solution of (0.1). Consequently, if $u$ is a nonnegative solution of (0.1) satisfying $u = 0$ on $∂Ω \setminus \{0\}$ then either $u ≡ 0$, or $u = u_{k,0}$ for some $k > 0$, or $u = u_{∞,0}$.

iii) When $q_c ≤ q ≤ 2$ isolated boundary singularities are removable, i.e. if $u ∈ C^2(Ω) \cap C(\overline{Ω} \setminus \{0\})$ is a nonnegative solution of (0.1) satisfying $u = 0$ on $∂Ω \setminus \{0\}$ then $u ≡ 0$. 