Isolated boundary singularities of positive solutions to quasilinear elliptic equations

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Abstract We study nonnegative solutions of the quasilinear equation

$$-\Delta u + |\nabla u|^q = 0 \qquad \text{in } \Omega \tag{0.1}$$

where $1 \leq q \leq 2$ and Ω is a C^2 bounded domain in $\mathbb{R}^N$ with $\partial\Omega$ containing the origin 0. We show that

i) When $q = 1$, for any $k > 0$, there exists a unique solution $u_{k,0}$ of (0.1) with boundary data $k\delta_0$ where δ_0 is the Dirac mass at 0. Moreover, $\lim_{k\to\infty} u_{k,0} = \infty$.

ii) When $1 < q < q_c = \frac{N+1}{N}$, for any $k > 0$, there exists a unique solution $u_{k,0}$ of (0.1) with boundary data $k\delta_0$. Moreover $u_{\infty,0} = \lim_{k\to\infty} u_{k,0}$ is a solution of (0.1). Consequently, if u is a nonnegative solution of (0.1) satisfying $u = 0$ on $\partial\Omega \setminus \{0\}$ then either $u \equiv 0$, or $u = u_{k,0}$ for some $k > 0$, or $u = u_{\infty,0}$.

iii) When $q_c \leq q \leq 2$ isolated boundary sinularities are removable, i.e. if $u \in C^2(\Omega) \cap C(\overline{\Omega} \setminus \{0\})$ is a nonnegative solution of (0.1) satisfying $u = 0$ on $\partial\Omega \setminus \{0\}$ then $u \equiv 0$.