Optimal Hardy-type inequalities

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ABSTRACT

We give a general answer to the following fundamental problem posed by Shmuel Agmon 30 years ago:

Given a second-order elliptic operator $P$ in $\mathbb{R}^n$, find a continuous, nonnegative weight function $W(x)$ which is “as large as possible” such that for some neighborhood of infinity $\Omega_R$ the following inequality holds

$$\langle P\phi, \phi \rangle \geq \int_{\Omega_R} W(x)|\phi|^2 \, dx \quad \forall \phi \in C_0^\infty(\Omega_R).$$

Our method is based on the theory of positive solutions and applies to both symmetric and nonsymmetric operators on a general domain $D$. The constructed weight is given by an explicit simple formula involving two positive solutions of the equation $Pu = 0$. This is a joint work with Baptiste Devyver and Martin Fraas.