The complete-lattice approach to set-valued optimization: Solution concepts, duality, optimality conditions, applications.

Abstract. Optimization problems with a set-valued objective have been paid some attention since the 1980ies, mainly motivated by duality issues in vector optimization (Corley, Luc and others). In combination with so-called set relations (Kuroiwa, Tanaka, Truong 1997), such problems could be treated like vector optimization problems: Instead of looking for minimal points with respect to a vector order, one could look for minimal sets with respect to a set relation, thus just lifting the major drawbacks of vector optimization (no infimum/supremum available) one level up.

The theory has been turned up-side-down by the complete-lattice approach (Hamel, Löhne): The set relations are used to construct appropriate image spaces for set-valued functions which turn out to share all properties with the extended reals essential for the development of an optimization theory, in particular convex duality.

In this talk, the basic constructions are introduced, and it is demonstrated that the theory of set-valued (convex) optimization is as complete as its scalar counterpart. In particular, we will discuss Fenchel conjugates and directional derivatives for set-valued functions and present some duality results. A new solution concept is given and corresponding optimality conditions are discussed.

Finally, we will show how the theory perfectly fits the needs of a particular application, namely models of financial markets with transaction costs or liquidity constraints: Set-valued convex duality can be used to derive super-hedging theorems and dual representation results for set-valued risk measures.

Some references.

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