On the minima of the functional Mahler product

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Abstract

If $\phi : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ if a convex function such that $0 < \int e^{\phi} < +\infty$, the *Mahler product* of ϕ is defined by

$$P(\phi) = \min_{z \in \mathbb{R}^n} \int e^{-\phi} \int e^{-\mathcal{L}^z \phi}$$

where $\mathcal{L}^z \phi(y) = \sup_{x \in \mathbb{R}^n} \langle x - z, y - z \rangle - \phi(x)$ is the Legendre trnsform of ϕ with respect to z. We prove that on the set of all convex functions (convex even functions) the Mahler product has no local minimum at any function (even function) having some regular point. This extends a recent result obtained by Reisner-Schuett-Werner for convex bodies.