

106928: Problem Set 6 due Wednesday, April 30

1. Let G be a topological graph drawn in the plane in such a way that every two edges cross an odd number of times. Show that every two odd cycles in G must share a common vertex.

suggestion: show that every two closed curves in the plane must cross an even number of times.

2. Show that there exists an antipodal geometric graph with n vertices and at least $cn^{4/3}$ edges (for some absolute constant $c > 0$).

suggestion: Start with a construction of n lines and n points that demonstrates $c'n^{4/3}$ incidences. Consider it as a geometric graph with vertices and edges. Now modify the drawing by possibly adding some new vertices of degree 1, and slightly changing the location of existing vertices.

3. Let P be a set of $n \geq 4$ points in general position in the plane. Assume that n is even and show that the number of $(\frac{n}{2} - 1)$ -edges of P is less than or equal to the number of $(\frac{n}{2} - 2)$ -edges of P .

suggestion: consider the allowable sequence associated with P .

4. Let \mathcal{F} be the family of all axes-parallel rectangles in the plane. That is, \mathcal{F} consists of all sets of the form

$$\{(x, y) \in \mathbb{R}^2 \mid a \leq x \leq b \text{ and } c \leq y \leq d\},$$

for all possible real parameters a, b, c, d .

Show that $VC - \dim(\mathcal{F}) = 4$.