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The multi-dimensional von Neumann alternating direction search algorithm in $C(B)$ and L_1.

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An n-dimensional subspace U of a normed linear space X is given, where U is spanned by m elements $u_1, \ldots, u_m$. To find the best approximation to a given element $f \in X$ from U, this paper gives an algorithm to solve this problem by cyclically searching the one- dimensional spaces spanned by each u_i $(i = 1, \ldots, m)$. The best approximation $u^{(k)}$ from the one-dimensional spaces is subtracted from f before searching in the next direction.

This algorithm is essentially the cyclic coordinate algorithm described by J. von Neumann or by *W. I. Zangwill* [Nonlinear programming. A unified approach (1969; Zbl 0195.208)], in special cases the Diliberto-Strauss algorithm is obtained.

Two questions are studied: Does the sequence $u^{(k)}$ converge and does it converge to a best approximation to f from U? There are some new results in so called smooth linear spaces X to characterize convergence. Detailed considerations of the authors' method are given in the spaces $C(B)$ and L_1, respectively.

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