

Zbl 1185.15028**Pinkus, Allan****Totally positive matrices.**

Cambridge Tracts in Mathematics 181. Cambridge: Cambridge University Press (ISBN 978-0-521-19408-2/hbk). xi, 182 p. £ 40.00; \$ 68.00 (2010).

The monograph deals with the theory of totally positive and oscillatory matrices. Basic definitions and statements of the finite-dimensional theory of total positivity are given in this monograph with all possible explicitness and accessibility, all the proofs given in the monograph are explicit and clear. The presentation of the material is strict and clear, this is an important merit of the given monograph.

The book consists of 6 chapters. Chapter 1 *Basic properties of totally positive and strictly totally positive matrices* is dedicated to the basic properties of this class of matrices. The basic definitions are given here as well as some basic properties and determinantal inequalities which are true for totally positive matrices. Chapter 2 *Criteria for total positivity and strictly total positivity* deals with general criteria of these properties and special classes of totally positive and strictly totally positive matrices; in particular, *LDU* factorizations of matrices are considered. The variation diminishing property (i.e., the statement that a finite dimensional totally positive operator does not enlarge the number of the sign changes of a vector, first introduced by I. J. Schoenberg) is analysed in Chapter 3 *Variation diminishing*. Chapter 4 *Examples* present different examples of totally positive matrices; in particular, here Cauchy matrices, Green matrices, Jacobi matrices, Hankel matrices, Toeplitz and Hurwitz matrices are presented. Chapter 5 *Eigenvalues and eigenvectors* is dedicated to oscillatory matrices and the famous Gantmacher-Krein theorem about the existence of n positive simple different in modulus from each other eigenvalues of an $n \times n$ oscillatory matrix; here the fundamental properties of eigenvectors of oscillatory matrices are presented. And the last chapter 6 *Factorizations of totally positive matrices* is dedicated to results on factorization of totally positive matrices.

There are two basic monographs dedicated to the theory of total positivity: “Oscillation matrices and kernels and small vibrations of mechanical systems” [Providence, RI: AMS Chelsea Publishing (2002; Zbl 1002.74002)] by *F. R. Gantmacher* and *M. G. Krein* and “Total positivity” [Stanford, California: Stanford University Press (1968; Zbl 0219.47030)] by *S. Karlin*. In both these monographs the main attention is given to the study of the infinite-dimensional case, or, more precisely, to linear integral operators with continuous totally positive kernels and different applications of totally positive operators. Moreover, in the monograph of *F. R. Gantmacher* and *M. G. Krein* most of the statements and proofs are given through the applications of the theory of total positivity to mechanics. So it is not always easy for the reader to find necessary information about the totally positive matrices. We remark that a concise survey on totally positive matrices and related topics was presented in the paper “Totally positive matrices” [Linear Algebra Appl. 90, 165–219 (1987; Zbl 0613.15014)] by *T. Ando*. Actually, as the author writes in the foreword, “Thankfully we have the short monograph of *T. Ando* . . . The present monograph is an attempt to update and expand upon the *Ando* monograph”. So, the pure theory of totally positive matrix is given in this small monograph.

We should mention also the usage in the book of exterior products, which makes some proofs easier and clearer (for example, of Kronecker’s theorem). Some recent results describing properties of totally positive matrices [for example, by *O. M. Katkova* and *A. M. Vishnyakova*, Linear Algebra Appl. 416, No. 2–3, 1083–1097 (2006; Zbl 1106.15014)], are also included in this monograph. But the aim of the monograph is not to give all the modern results on new applications of the theory of positivity, but the strict and clear presentation of the basic facts, so an enormous number of recent results are excluded from the “Totally positive matrices”. Unfortunately, the cone-theoretical approach to the theory of totally positive matrices, developed in the papers of *M. A. Krasnosel’skiĭ* and *A. V. Sobolev*, is not mentioned in this monograph either.

The monograph contains an extended list of references, which gives the reader an opportunity to learn not only the classical results, but also some modern results, excluded from the main presentation. So, to summarize, we should say that the monograph “Totally positive matrices” by *A. Pinkus* is an ideal book for the first acquaintance with the theory of total positivity.

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