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TDI-subspaces of $C(\mathbb{R}^d)$ and some density problems from neural networks.

Space $W \subset C(\mathbb{R}^d)$ is said to be TDI if it is invariant under actions of the group generated by translations and dilations (in each coordinates) of $\mathbb{R}^d$. Let $M_f$ be the closure of the smallest TDI-space which contains $f$.

Theorem 1. $M_f \neq C(\mathbb{R}^d) \iff$ there is $\alpha \in \mathbb{Z}_d^+$ such that $D^\alpha f = 0$ (in the weak sense). Let $A_\alpha := \text{span}\{x^\beta; \beta \leq \alpha\}$; here some coordinates of $\alpha$ may be infinite but all $\beta \in \mathbb{Z}_d^+$. Theorem 2. Every TDI-space is a finite sum of the $A_\alpha$.

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