Zentralblatt MATH

Zbl 1222.41045

Pinchasi, Rom; Pinkus, Allan

Dominating subsets under projections.

SIAM J. Discrete Math. 24, No. 3, 910-920 (2010).

Let W be a d-dimensional subspace of the space ℓ_1^n, that is the space $\mathbb{R}^n$ equipped with the ℓ^1-norm $\|x\| = \sum_{i=1}^n |x_i|$, for $x = (x_1, \dots, x_n) \in \mathbb{R}^n$. Let $\operatorname{supp}(x) = \{i : x_i \neq 0\}$ and let $I \subset \{1, 2, \dots, n\}$. Using characterization results for best ℓ^1-approximation (see, for instance [*A. M. Pinkus*, On L^1-approximation. Cambridge: Cambridge University Press (1989; Zbl 0679.41022)]), it follows that the vector 0 is a best ℓ^1-approximation in W to $x \in \mathbb{R}^n$ with $\operatorname{supp}(x) \subset I$ if and only if (1) $\sum_{i\in I} |w_i| \le \sum_{i\notin I} |w_i|$, for all $w \in W$. The paper is concerned with the existence of such a set I and, if so, how large it can be. A simple lower bound is given in Proposition 1: there exists such a set I with $|I| \ge [n/(d+1)]$ (here $[\alpha]$ stands for the integer part of a number $\alpha \in \mathbb{R}$).

The main result of the paper (Theorem 1) asserts that for any d-dimensional subspace W of $\mathbb{R}^n$ there exists $I \subset \{1, 2, \dots, n\}$ of cardinality $(2^{-1} - o(1))n$ for which (1) holds. The proof is based on an equivalent dual form of Theorem 1: If P is a subset of cardinality n of $\mathbb{R}^d$, then there exists a subset S of P of cardinality at least $(2^{-1} - o(1))n$ such that $\sum_{x\in S} |\langle x, u\rangle| \le \sum_{x\in P\setminus S} |\langle x, u\rangle|$ for all $u \in \mathbb{R}^d$.

In the last part of the paper one discusses the analog of this problem in the space $C[0,1]$ equipped with the L^1-norm. For instance, for every $\varepsilon > 0$ there exists a subset K of $[0,1]$ of Lebesgue measure at least $2^{-1} - \varepsilon$ such that (2) $\int_K |f| \le \int_{[0,1]\setminus K} |f|$, for every $f \in W$, where W is a given finite-dimensional subspace of $C[0,1]$ (Theorem 5). One shows also that if W contains the quadratic polynomials, then one cannot get rid of $\varepsilon > 0$ in (2), but for bidimensional subspaces of the form $W = \operatorname{span}\{1, g\}$ with $g \in C[0,1]$ picewise monotone, one obtains exactness, i.e. there exists a set $K \subset [0,1]$ of measure 2^{-1} for which (2) holds.

Stefan Cobzaş (Cluj-Napoca)

Classification: 41A63 41A45 41A50

Keywords: best approximation; discrepancy; L^1-approximation

doi:10.1137/090781164