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Uniqueness in vector-valued approximation.

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By vector-valued functions one means the mappings $f(x) := (f_1(x), \dots, f_m(x))$, $x \in D$, where D is some set and each f_i $(i = 1, \dots, m)$ is defined on D with real values. One can consider the mixed norms of such functions given by

$$\|f\|_{A(p,q)} := \left(\sum_{i=1}^{m} \left(\int_D |f_i(x)|^q d\nu(x) \right)^{p/q} \right)^{1/p}$$

and

$$\|f\|_{B(p,q)} := \left(\int_D \left(\sum_{i=1}^{m} |f_i(x)|^q \right)^{p/q} d\nu(x) \right)^{1/p}$$

where $p, q \in [1, \infty]$. The author gives in the main an answer at the following question:
Given a finite dimensional subspace U, what are conditions on U such that to each f there exists a unique best approximant from U in the norm $\| \cdot \|_{A(p,q)}$ or $\| \cdot \|_{B(p,q)}$? The paper contains three sections. After an introductory paragraph, he gives a series of known results concerning characterizing best approximations and uniqueness. These results are used in Section 3 which presents a review of some results on unicity spaces in the C and L^1 norms, as these are the basic non-smooth, non-strictly convex norms considered. Sections 4-12 represent Part A and are concerned with $A(p, q)$-norms and Sections 13-19 represent Part B and deal with the $B(p, q)$-norms. *S.S.Dragomir (Timişoara)*

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