A problem of approximation using multivariate polynomials.

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The authors are concerned with the approximability of multivariate functions by simple ones. As an example, Kolmogorov’s theorem is quoted, though this is entirely nonconstructive. A reason for this is discussed. In this article, simple functions are of the form \( f(h(\cdot)), f \in C(\mathbb{R}), h \) varying over all shifts and/or dilations of a fixed multivariate polynomial \( g \). If \( G_{1,2,3} \) are the spans of the sets of all shifted, dilated or both shifted and dilated polynomials originating from \( g \), respectively, then \( G_i := \text{span}\{f(h(\cdot)) : h \in G_i, f \in C(\mathbb{R})\} \) for \( i = 1, 2, 3 \), and density questions are discussed for these spaces. One typical result is the following (Theorem 3.9): Let \( g \) denote any 3-variate polynomial. Then \( \mathcal{P}(g) := \text{span}\{(g(\cdot - b))^k : b \in \mathbb{R}^3, k \in \mathbb{Z}_+\} \) is the full space of 3-variate polynomials if and only if \( \mathcal{P}(g) \) separates points. There are more results of this type, also for higher dimensions but most of nonconstructive type.

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