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A problem of approximation using multivariate polynomials.

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The authors are concerned with the approximability of multivariate functions by simple ones. As an example, Kolmogorov's theorem is quoted, though this is entirely nonconstructive. A reason for this is discussed. In this article, simple functions are of the form $f(h(\cdot))$, $f \in C(\mathbb{R})$, h varying over all shifts and/or dilations of a fixed multivariate polynomial g. If $G_{1,2,3}$ are the spans of the sets of all shifted, dilated or both shifted and dilated polynomials originating from g, respectively, then $\mathcal{G}_i := \operatorname{span}\{f(h(\cdot)) : h \in G_i,\ f \in C(\mathbb{R})\}$ for $i = 1, 2, 3$, and density questions are discussed for these spaces. One typical result is the following (Theorem 3.9): Let g denote any 3-variate polynomial. Then $\mathcal{P}(g) := \operatorname{span}\{(g(\cdot - b))^k : b \in \mathbb{R}^3, k \in \mathbb{Z}_+\}$ is the full space of 3-variate polynomials if and only if $\mathcal{P}(g)$ separates points. There are more results of this type, also for higher dimensions but most of nonconstructive type. *M.Reimer (Dortmund)*

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