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Smoothest interpolants.

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This note is in the "Open Problems"section of the journal. The author gives the history of the following extremal problem: Given m fixed points $0 \le t_1 < \cdots < t_m$, m real data $e_1, \ldots, e_m$, and a natural number $n \le m$, find

$$\inf\{\|f^{(n)}\|_p : f \in W_p^n[0,1],\ f(t_i) = e_i,\ i = 1, \ldots, n\}$$

and characterize the extremal f. Here, $\|\cdot\|_p$ is the usual $L_p[0,1]$ norm, $1 < p \le \infty$, and $W_p^n[0,1]$ is the standard Sobolev space. The special cases $p = 2$ and $p = \infty$ played an important role in the development of spline theory. For $1 < p < \infty$ the extremal function is characterized by having an n-th derivative of the form

$$f^{(n)}(x) = |h(x)|^{-1}\mathrm{sgn}(h(x)),$$

where $1/p + 1/q = 1$ and h is a spline function of degree $n - 1$ with simple knots $t_1, \ldots, t_m$, which vanishes identically outside $[t_1, t_m]$.

A generalization of the above problem is the following: Keeping the data (e_i) fixed, find the infimum of $\|f^{(n)}\|_p$ over $W_p^n[0,1]$, where the interpolation points (t_i) are allowed to vary in [0,1] maintaining their order. A motivation for this generalization comes from certain geometric and physical interpretation. Under a certain assumption for the data, it turns out that the extremal function f possesses the same characteristic property, and, in addition, the extremal interpolation nodes are also points of local extrema for f.

The more difficult part in the second problem is the uniqueness. In fact, the uniqueness of the smoothest interpolant is known only in the following cases: all n, $p = \infty$; $n = 2$ and all $p \in (1, \infty)$; $n = 2$, $p = 3$, and in the multidimensional case, for $p = 2$ and all n. Generally, the uniqueness is still an open problem.

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