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Some remarks on nonnegative polynomials on polyhedra.

Probability, statistics, and mathematics, Pap. in Honor of Samuel Karlin, 163-186 (1989).

[For the entire collection see Zbl 0679.00013.]

A well-known theorem of Lukács and Szegö says that each algebraic polynomial p(t) of degree at most n, nonnegative on [a,b] has a representation of the form

$$(1) \quad p(t)=\begin{cases} (t-a)(t-b)R^2(t)+Q^2(t) & (n=2m) \\ (t-a)S^2(t)+(b-t)T^2(t) & (n=2m-1), \end{cases}$$

where R, S and T are real polynomials of degree at most m-1, Q of degree at most m. A refinement of this result is due to *S. Karlin* and *L. S. Shapley* [Mem. Am. Math. Soc. 12, 93 p. (1953; Zbl 0052.185)].

In connection with these results the authors give a similar representation for quadratic polynomials of two variables on a triangle in $\mathbb{R}^2$. They prove an analogue of (1) for nonnegative quadratic polynomials on simplices in $\mathbb{R}^3$ and show that an analogue of (1) and the representation of Karlin and Shapley on simplices in $\mathbb{R}^4$ are in general false. *T.Šalát*

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