Zbl 0706.26013
Micchelli, Charles A.; Pinkus, Allan
Some remarks on nonnegative polynomials on polyhedra.
[For the entire collection see Zbl 0679.00013.]
A well-known theorem of Lukács and Szegő says that each algebraic polynomial \( p(t) \) of degree at most \( n \), nonnegative on \([a,b]\) has a representation of the form

\[
(1) \quad p(t) = \begin{cases} 
(t-a)(t-b)R^2(t) + Q^2(t) & (n = 2m) \\
(t-a)S^2(t) + (b-t)T^2(t) & (n = 2m - 1)
\end{cases}
\]

where \( R, S \) and \( T \) are real polynomials of degree at most \( m-1 \), \( Q \) of degree at most \( m \). A refinement of this result is due to S. Karlin and L. S. Shapley [Mem. Am. Math. Soc. 12, 93 p. (1953; Zbl 0052.185)].

In connection with these results the authors give a similar representation for quadratic polynomials of two variables on a triangle in \( \mathbb{R}^2 \). They prove an analogue of (1) for nonnegative quadratic polynomials on simplices in \( \mathbb{R}^3 \) and show that an analogue of (1) and the representation of Karlin and Shapley on simplices in \( \mathbb{R}^4 \) are in general false.

T.Šalát

Classification: 26C99 41A63
Keywords: polyhedra; Bernstein-Bezier form; representation; polynomials