Zbl 0551.41001

Pinkus, Allan

n-widths in approximation theory.

Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge, Bd. 7. Berlin etc.: Springer-Verlag. X, 291 p. DM 118.00 (1985).

Let X be a normed linear space and A a given subset of X. As a measure of how the "worst element" of A can be approximated by "finite dimensional" objects, the following numbers are considered: (1) $d_n(A,x) = \inf\{\sup_{x\in A}\inf_{y\in X_n}\|x-y\|;\ X_n$ is an n- dimensional subspace of $X\}$, (2) $\delta_n(A,x) = \inf\{\sup_{x\in A}\|x-P_n(x)\|;$ P_n is a continuous linear operator of rank $\le n\}$, (3) $d^n(A,x) = \inf\{\sup_{x\in A\cap L^n}\|x\|;\ L^n$ is a subspace of X of codimension $n\}$. These are called - respectively - the Kolmogorov n-width, the linear n-width, the Gel'fand n width of A in X. They are interesting in approximation theory, and it is interesting to have estimates for $n\to\infty$, to know existence of operators or subspaces for which the infimum is obtained, and so on. Other n-widths appear in the literature, but they are not considered here.

Several relations exist among the three kinds of width indicated above. The topics discussed are rather yound (among the 240 references only a few of them are more than 20 years old); so the book is claimed to be not a definitive one. Anyway, it presents a good deal of material, with a small preference for exact results (rather than for asymptotic estimates). After the introduction (chapter I), there is a theoretical chapter concerning general propeties of the n-widths and relationships among them (chapter II). Chapter III is concerned with results concerning Chebyshev systems and total positivity, which are extensively used in the following chapters. In Chapter IV n-widths in Hilbert spaces are considered. Chapters V and VII deal with Sobolev spaces (exact n-widths and asymptotic estimates, respectively). Chapter IV deals with matrices and n-widths, while in Chapter VIII various classes of analytic functions are considered. On the whole, a very good book on an expanding subject. *P.L.Papini*

Classification: 41-02 41A46 41A65
Keywords: Kolmogorov n-width; linear n-width; Gel'fand n width; Chebyshev systems; total positivity; Hilbert spaces; Sobolev spaces; analytic functions