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Fundamentality of ridge functions.
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For $d, n \in \mathbb{N}$ denote by Ω the set of all real $d\times n$ matrices and let $M(\Omega) = \operatorname{span}(g(A.x)\colon\ A \in \Omega,\, g \in C(\mathbb{R}^d, \mathbb{R})\}$, where $C(\mathbb{R}^d, \mathbb{R})$ denotes the space of all continuous functions $g : \mathbb{R}^d \to \mathbb{R}$ For $d = 1$ the functions in $M(\Omega)$ are called ridge functions and they are relevant in problems of tomography, neural networks, approximation theory etc. The main question treated in this paper is the density of $M(\Omega)$ in $C(\mathbb{R}^n, \mathbb{R})$, with respect to the topology of uniform convergence of compacta. For $A \in \Omega$ let $L(A)$ be the span of the d rows of A and let $L(\Omega) = \bigcup\{L(A)\colon\ A \in \Omega\}$. Let H^k_n denote the set of homogeneous polynomials p of n variables

$$-p(s) = \sum_{|m|=k} c_m s^m, \qquad m = (m_1, \dots, m_n) \in \mathbb{Z}^n,$$

$|m| = m_1 + \dots + m_n$, $s^m = s_1^{m_1} \dots s_n^{m_n}$, and let $H^n = \bigcup_{n=0^\infty} H^n_k$. Then $M(\Omega)$ is dense in $C(\mathbb{R}^n, \mathbb{R})$ iff the only polynomial in H^n vanishing identically on $L(\Omega)$ is the zero polynomial (Theorem 2.1). This is equivalent to the assertion that $M(\Omega)$ explicitly contains the polynomials. If $k \in \mathbb{N}$ is fixed, then

$$M_k(\Omega) = \left\{ \sum_{i=1}^{k} a_i g(A_i x) :\ a_i \in \mathbb{R},\ A_i \in \Omega,\ i = 1, \dots, k,\ g \in C(\mathbb{R}^d, \mathbb{R}) \right\}$$

is not dense in $C(\mathbb{R}^d, \mathbb{R})$ (Theorem 5.1). Theorems 2.1 and 4.1 in this paper are obtained (in the case $d = 1$) by *B. A. Vostrecov* and *M. A. Krejnes*, Dokl. Akad. Nauk SSSR 140, 1237-1240 (1961; Zbl 0106.273) and ibid. 144, 1212-1214 (1962; Zbl 0196.439). *S.Cobzaş (Cluj-Napoca)*

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