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Nonlinear eigenvalue-eigenvector problems for STP matrices.
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A matrix A is strictly totally positive (STP) if all its minors are positive. A theorem due to *F. Gantmakher* and *M. Krein* [Compositio Math. 4, 445-476 (1937; Zbl 0017.00102)] states that a matrix of this class of order N has all its N eigenvalues positive and simple. Moreover the signs of the components of each eigenvector have some properties. In this paper the authors obtain some generalizations of this result. For a vector $\mathbf{x} = (x_1, \ldots, x_N) \in R^N$ and arbitrary $p > 0$ they define

$$\mathbf{x}^{p^*} = (|x_1|^p \operatorname{sgn} x_1, \ldots, |x_N|^p \operatorname{sgn} x_N).$$

Then they consider the eigenvalue-eigenvector problem

(1) $$A_m(\cdots(A_2(A_1x)^{p_1^*})^{p_2^*}\cdots)^{p_{m-1}^*} = \lambda x^{r^*},$$

where the $p_1, \ldots, p_{m-1}, r$ are arbitrary positive numbers and A_i is an $N_i \times N_{i-1}$ STP matrix with $N_0 = N_m = N$. The main result of the paper states that for $p_1, \ldots, p_{m-1} \geq 1$ and $r = p_1 p_2 \cdots p_{m-1}$ there exist exactly $R = \min\{N_i : i = 1, \ldots, m\}$ eigenvalue-eigenvector pairs (λ_i, x_i) satisfying (1). Furthermore $\lambda_1 > \cdots \lambda_R > 0$, the components of x_i present exactly $i - 1$ sign changes and every zero component if it exists, is flanked by nonzero components of opposite sign. This result also generalizes a previous one by *A. Pinkus* [Linear Algebra Appl. 64, 141-156 (1985; Zbl 0553.15008)]. *Mariano Gasca (Zaragoza)*

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