Nonlinear eigenvalue problems for a class of ordinary differential equations.

Here, nonlinear eigenvalue problems of the form

\[(a_{n-1}(\cdots(a_1((a_0u^{p_0})')^{p_1})')^{p_2})\cdots')^{p_{n-1}}') = \lambda bu^{p_n}\]

on a compact interval \([a,b]\) subject to a class of separated boundary conditions (of which \(k\) are imposed at \(a\)) are studied. The main result is that, if 0 is not an eigenvalue and the exponents \(p_i\) satisfy a certain condition, then the eigenvalues of such a problem form an infinite sequence of real numbers \(\lambda_i\) which are positive and tend to \(\infty\) if \(n-k\) is even, otherwise, they are negative and tend to \(-\infty\). Further it is shown, in analogy to Sturm-Liouville theory, that to each \(\lambda_i\) there corresponds an essentially unique eigenfunction \(u_i\) that has precisely \(i-1\) zeros in \((a,b)\), and the zeros of \(u_i\) and \(u_{i-1}\) in \((a,b)\) interlace.

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