Several easily verified conditions on a sequence \((c_k)_1^\infty\) of real numbers are given which imply that the sequence of functions \((x^k - c_k)_1^\infty\) is total in \(C[0,1]\). This problem is equivalent to demanding that the function \(f(x) \equiv 1\) belongs to the closed linear hull of \((x^k - c_k)_1^\infty\) in \(C[0,1]\). For instance, if the sequence \((c_k)_1^\infty\) is such that for all \(k \geq M\), \(\epsilon(-1)^k(c_k - c) \geq 0\), where \(c \in \mathbb{R}\) and \(\epsilon \in \{-1,1\}\), fixed, and if \(c_k - c \neq 0\), then \((x^k - c_k)_1^\infty\) is total in \(C[0,1]\); if, in addition, \(c_k \neq c\) for infinitely many \(k\), with the help of Chebyshev polynomials an effective approximation to \(f(x) \equiv 1\) in \(C[0,1]\) by finite linear combinations of the \(x^k - c_k\) is given. Another condition is: \(|c_{n_k} - c|^{1/n_k} \to 0\) as \(k \to \infty\), where the subsequence \((n_k)_1^\infty\) satisfies the Müntz condition \(\sum_{k=1}^{\infty} (n_k)^{-1} = \infty\) and \(c_k \neq c\); in the case when \(|c_k|^{1/k} \to 0\) as \(k \to \infty\), again, a good approximation to \(f(x) \equiv 1\) is explicitly constructed.

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