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Hershkowitz, Daniel; Pinkus, Allan

On nonnegative sign equivalent and sign similar factorizations of matrices.

Electron. J. Linear Algebra 16, 162-170, electronic only (2007).

The following definitions are used in the paper: A real square matrix A is said to be totally positive/strong totally positive if all its minors are nonnegative/positive; an upper triangular matrix A is said to be triangular strictly totally positive if all its minors that can be possibly nonzero are positive. A matrix A is said to be nonnegative sign equivalent/positive sign equivalent if it can be factorized in the form $A = D_1QD_2$ with Q (entrywise) nonnegative/totally positive and D_1 and D_2 diagonal matrices with diagonal elements equal to ± 1; if in the first case $D_1 = D_2$, A is said to be nonnegative sign similar.

The main result states that any real square matrix A can be factorized in the form $A = DQB$ where D is a diagonal matrix with diagonal elements equal to ± 1, Q is a nonnegative matrix and B is the inverse of an upper triangular strictly totally positive matrix with diagonal elements equal to 1. As a corollary it is shown that every real square matrix is a product of at most two nonnegative sign equivalent matrices, It is also shown that every real square matrix is a product of at most three nonnegative sign similar matrices.

Another result states that every real square matrix is a product of some totally positive sign equivalent matrices. The problem of the minimal number of these factors is stated as an open question.

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