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On nonnegative sign equivalent and sign similar factorizations of matrices.

The following definitions are used in the paper: A real square matrix $A$ is said to be totally positive/strong totally positive if all its minors are nonnegative/positive; an upper triangular matrix $A$ is said to be triangular strictly totally positive if all its minors that can be possibly nonzero are positive. A matrix $A$ is said to be nonnegative sign equivalent/positive sign equivalent if it can be factorized in the form $A = D_1QD_2$ with $Q$ (entrywise) nonnegative/totally positive and $D_1$ and $D_2$ diagonal matrices with diagonal elements equal to $\pm 1$; if in the first case $D_1 = D_2$, $A$ is said to be nonnegative sign similar.

The main result states that any real square matrix $A$ can be factorized in the form $A = DQB$ where $D$ is a diagonal matrix with diagonal elements equal to $\pm 1$, $Q$ is a nonnegative matrix and $B$ is the inverse of an upper triangular strictly totally positive matrix with diagonal elements equal to 1. As a corollary it is shown that every real square matrix is a product of at most two nonnegative sign equivalent matrices. It is also shown that every real square matrix is a product of at most three nonnegative sign similar matrices. Another result states that every real square matrix is a product of some totally positive sign equivalent matrices. The problem of the minimal number of these factors is stated as an open question.

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