Zbl 0642.41020
Pinkus, Allan
Continuous selections for the metric projection on $C_1$.

Let $U$ be a finite dimensional subspace of $C_1(K)$, the continuous functions normed with the $L_1$ norm. $K$ is a compact subset of $R^n$ and equal the closure of its interior. This paper proves that there is a continuous selection for the metric projection onto $U$ only in the trivial case that $U$ is a Chebyshev set. This tidy theorem for such a basic setting well supplements the few similar results in the literature for other spaces. In fact it is shown that if $K$ is connected there does not exist a selection, $s$, for the metric projection such that even $f_n$ converging uniformly to $f$ does not mandate the convergence of $s(f_n)$ to $s(f)$. The proofs involve a series of lemmas manipulating the characterization of best approximations in $C_1(K)$ and the substructures forced upon $U$ by the existence of a continuous selection for the metric projection. 

D.Wulbert

Classification: 41A50 41A65
Keywords: metric projection; Chebyshev set; continuous selection
doi:10.1007/BF02075449