As a partial generalization of the Bernstein comparison theorem, D. Braess [Aequationes Math. 12, 80-81 (1975; Zbl 0328.41012)] proved the theorem: Let $f, g \in C^{n+1}[a, b]$ and assume that

$$0 \leq g^{(n+1)}(x) \leq f^{(n+1)}(x), \quad x \in [a, b].$$

Then

$$\min_{s \in S_{n,k}} \|g - s\|_\infty \leq \min_{s \in S_{n,k}} \|f - s\|_\infty.$$  

The author extended this theorem to the case where

$$|g^{(n+1)}(x)| \leq f^{(n+1)}(x), \quad x \in [a, b]$$

replaces assumption (1). In this case (2) is no longer valid. He demonstrates this fact by determining the precise upper bound on the best approximation of the functions $g$ satisfying (3), by splines of degree $n$ with $k$ variable knots. It is also shown that interpolation at $n + k + 1$ fixed, distinct points by splines of degree $n$ with $k$ fixed knots (these depend upon $f$) gives the same upper bound as that which is obtained for the best approximation from $S_{n,k}$ to the class of functions $g$ satisfying (3). The author also characterizes the best constant in the degree of approximation from $S_{n,k}$ of functions in the Sobolev space $W^{n+1}_\infty$ which satisfy $|g^{(n+1)}|_\infty \leq 1$.

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