Zbl 0533.41021

Pinkus, A.

Bernstein's comparison theorem and a problem of Braess.

Aequationes Math. 23, 98-107 (1981).

As a partial generalization of the Bernstein comparison theorem, *D. Braess* [Aequationes Math. 12, 80-81 (1975; Zbl 0328.41012)] proved the theorem: Let $f, g \in C^{n+1}[a,b]$ and assume that (1) $0 \leq g^{(n+1)}(x) \leq f^{n+1}(x)$, $x \in [a,b]$. Then (2) $\min_{s\in\mathcal{S}_{n,k}} \|g-s\|_\infty \leq \min_{s\in\mathcal{S}_{n,k}} \|f-s\|_\infty$. The author extended this theorem to the case where (3) $|g^{(n+1)}(x)| \leq f^{(n+1)}(x)$, $x \in [a,b]$ replaces assumption (1). In this case (2) is no longer valid. He demonstrates this fact by determining the precise upper bound on the best approximation of the functions g satisfying (3), by splines of degree n with k variable knots. It is also shown that interpolation at $n+k+1$ fixed, distinct points by splines of degree n with k fixed knots (these depend upon f) gives the same upper bound as that which is obtained for the best approximation from $\mathcal{S}_{n,k}$ to the class of functions g satisfying (3). The author also characterizes the best constant in the degree of approximation from $\mathcal{S}_{n,k}$, of functions in the Sobolev space ${W_\infty}^{n+1}$ which satisfy $|g^{(n+1)}|_\infty \leq 1$. *L.Leindler*

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