

Herman Müntz: A Mathematician's Odyssey

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Introduction

In 1885 Weierstrass [1] proved that every continuous function on a finite interval can be uniformly approximated by algebraic polynomials. In other words, algebraic polynomials are dense in $C[a,b]$ (for any $-\infty < a < b < +\infty$). This is a theorem of major importance in mathematical analysis and a foundation for approximation theory.

One of the first outstanding generalizations of the Weierstrass Theorem is due to Ch. H. Müntz, who answered a conjecture posed by S. N. Bernstein in a paper [2] in the proceedings of the 1912 *International Congress of Mathematicians* held at Cambridge, and in his 1912 prize-winning essay [3]. Bernstein asked for exact conditions on an increasing sequence of positive exponents α_n , so that the system $\{x^{\alpha_n}\}_{n=0}^{\infty}$ is complete in the space $C[0,1]$. Bernstein himself had obtained some partial results. On p. 264 of [2] Bernstein wrote the following: "It will be interesting to know if the condition that the series $\sum 1/\alpha_n$ diverges is necessary and sufficient for the sequence of powers $\{x^{\alpha_n}\}_{n=0}^{\infty}$ to be complete; it is not certain, however, that a condition of this nature should necessarily exist."

It was just two years later that Müntz [M7] was able to provide a solution confirming Bernstein's qualified guess. What Müntz proved is the following:

Theorem. *The system $\{x^{\alpha_0}, x^{\alpha_1}, x^{\alpha_2}, \dots\}$, where $0 \leq \alpha_0 < \alpha_1 < \alpha_2 < \dots$, is complete in $C[0,1]$ if and only if $\alpha_0 = 0$ and*

$$\sum_{n=1}^{\infty} \frac{1}{\alpha_n} = \infty.$$

Today there are numerous proofs and generalizations of this theorem, widely known as the "Müntz Theorem." In fact a quick glance at *Mathematical Reviews*, that is, at papers from 1940, shows nearly 150 papers with the name Müntz in the title. All these articles mention Müntz's name in reference to the above theorem, except one referring to his thesis. Müntz's name with his theorem appears in numerous books and papers. In addition there are Müntz polynomials, Müntz spaces, Müntz systems, Müntz type problems, Müntz series, Müntz-Jackson Theorems, and Müntz-Laguerre filters. The Müntz Theorem is at the heart of the Tau Method and the Chebyshev-like techniques introduced by Cornelius Lanczos [4]. In other words, Müntz has come the closest a mathematician can get to attaining a little piece of immortality.

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Notwithstanding, a quick search of the mathematical literature will also show that essentially nothing is known about Müntz, the person and the mathematician. The purpose of this paper is to try to redress this oversight. Müntz's life, mathematically and otherwise, makes for an illuminating and dramatic journey through the first half of the twentieth century. It is unfortunate it was not a more pleasant journey.

Early Years (1884-1914)

Herman Müntz³ (officially named Chaim) was born in Lodz on August 28, 1884. Müntz's family was bourgeois and Jewish, though not religious. At that time Lodz was a part of "Congress Poland" under Russian rule. It was an important industrial city at the western boundary of this area. In the last decades of the nineteenth century, when Müntz was born, it had a vibrant Jewish community, mainly engaged in textiles and other related trades, as well as in business in general. In official documents, Müntz's father is described as "in trade," with the suggestion that he was an estate agent. The family name was spelt in the German manner rather than the more common Minc. Herman was the eldest of five children, all of whom (except for the youngest) were sent to study at German and Swiss universities. The turbulent economic times were such that the family was generally, though not always, comfortably well off. A noticeable decline was associated with the depression of the late 1920's. Müntz started his studies at the *Höhere Gewerbeschule* in Lodz, the top technical high school in that city, with a bias toward textiles, textile machinery and chemistry. He was fluent in Polish and had a reasonably good command of German and Russian.

In 1902 Müntz went to Berlin to study at the Friedrich-Wilhelms-Universität, generally referred to as the University of Berlin (called Humboldt Universität Berlin since 1948), where he studied mathematics, the natural sciences and philosophy. In 1906 he earned his matriculation degree. He named Frobenius, Knoblauch, Landau, Schottky and Schwarz as his teachers, singling out Frobenius and Schwarz as his main influences.

From 1906 to 1910 Müntz was in Berlin where he worked, wrote and studied. In 1912 he married Magdalena (Magda) Wohlman who was from the area of Zlotkow near Poznan, an area of Poland under German control. Magda had come to Berlin to study biology. While the marriage would remain childless it was, by all accounts, an unusually harmonious union. During this early period Müntz was involved in the private teaching of mathematics. Money was always a pressing problem. For much of his life Müntz remained engaged in pedagogy in one form or another "teaching elementary and higher mathematics, partly in private schools and partly as a private tutor."⁴

Müntz was an intellectual who was intensely interested in philosophy, poetry, art and music. He was especially taken with Goethe but, more particularly, with Nietzsche's philosophy, which was to have a profound influence on him. He attended university lectures given by the philosopher Alois Riehl, and he seems to have written a thesis on Nietzsche.

In these years he also became interested in a reassessment of Jewish culture and the position of Jews in society. In 1907 he published a 124 page book called *Wir Juden* [6] in which the influence of Nietzsche, and especially his *Also sprach Zarathustra*, is discernible. The book,

³ The file on Müntz preserved at the *Society for the Protection of Science and Learning*, now at the Bodleian Library, Oxford, provided a valuable start in the search for other sources included. On the Müntz's files there see Ortiz [5].

⁴ Müntz to Geheeb, March 1, 1914. Archive of the École d'Humanité.

dedicated to Friedrich Nietzsche, is about the need for a basic reform of the Jewish people in the post-orthodox religious period, and a reconsideration of the position of Jews in contemporary society.

Müntz discussed, in detail, what he called the “new Jew,” the contribution Jewish people had made and could make to humanity, and characterized Jews not as a pure race but as a diversity of many peoples emphasizing the past and present connections between Jews and a variety of other people. The book also aspired to help the young Jewish generation of the time to achieve its religious and political self-definition. It embraced a view of Zionism, not uncommon at the time, in which socialist viewpoints were discernible. At a time when racism and anti-Semitism were rampant there were remarks in Müntz’s text that are very much race-based. This makes for a book very discomfiting to read today. The book was advertised in the Berlin Jewish/Zionist weekly “Jüdische Rundschau” in its list of “Zionistische Literatur.” These advertisements continually misspelt the author’s name as “Müntzer,” which might well be considered as a measure of the perceived importance of the book.

Aside from a mathematical text mentioned later, this was the only book by Müntz that was ever published. However, we have found various items of correspondence indicating that he also wrote at least three other (non-mathematical) texts. All written from about 1911 to 1924, they were: a) *Über Ehe und Treue* (On marriage and fidelity); b) A book about *Psalms*, and c) *Der Jüdische Staat* (The Jewish state). The three manuscripts were sent to different publishers, but for a variety of reasons, including the war and lack of paper, none seems to have been published. However, parts of the last-named book appeared as articles in a journal.

Despite these varied activities, Müntz’s main goal in the period 1906-1910 seems to have been his mathematical studies, which were under the supervision of Hermann Amandus Schwarz. His first results were of a geometric character, having to do with rational tetrahedra. However he soon began to produce results on the main topic of his doctoral dissertation, namely minimal surfaces defined by closed curves in space that mathematically involved the approximate solution of non-linear partial differential equations. On October 1, 1910, Müntz was awarded a doctorate, Dr. Phil., *magna cum laude*. His official reviewers were Schwarz and Schottky. His dissertation, under the title of “On boundary value problems of partial differential equations of minimal surfaces” was published in *Crelle*’s journal [M1]. This work is still occasionally referenced.

In this thesis Müntz studied the Plateau problem in some detail. He used potential theory and the method of successive approximation, two tools he would return to in subsequent papers. When Müntz was near the end of this dissertation work, Schwarz advised him that Arthur Korn, who was working in the same area, had submitted for publication a paper on the subject of his thesis, which was later published [7]. In his *Crelle* paper Müntz acknowledged Korn’s work. Although their results had a common ground, the techniques used by each author and the final results were sufficiently diverse to merit independent publication. Müntz seems to have been the last of Schwarz’s doctoral students. Other doctoral students of Schwarz included Leopold Fejér, Ernst Zermelo, Paul Koebe, and Leon Lichtenstein. The latter became a close friend of Müntz.

In late 1911 Müntz went to Munich to give a lecture at the seminar of Ferdinand von Lindemann. He was also accepted into Aurel Voss’s circle. These were two of the three mathematics professors at the Karl Ludwig-Maximilians Universität, in Munich, the third was Alfred Pringsheim. The Müntzes decided to move to Munich primarily on the basis of

this visit, which seemed to open some opportunities. But they were also undoubtedly influenced by the fact that two of Müntz's brothers were also then residing in Munich. However, Müntz's aim and that of any young aspiring mathematician in Germany at this stage of his career, was to secure a position as a "Privatdozent." The next stage was to gain a Habilitation and eventually an academic position at a university. At that time, and the same is essentially true today, the Habilitation was necessary for a professorship, and a professorship is what Müntz wanted then and throughout his life. According to his correspondence Müntz, who was not the only candidate, obtained the support of the three mathematics professors. It seems, however, that there were also what he termed some "strange regulations," and serious formal problems. The matter dragged on, seemingly interminably, but to no avail. In the end, Müntz was unsuccessful in gaining the dozent position.

While in Munich Müntz was again earning his living privately as a teacher at various levels. His wife also worked part-time and there was some financial help from the family. Müntz also attended lectures and seminars given by von Lindemann and Voss and was actively engaged in mathematics research. From 1912 to 1914 he published four papers on problems in the field of modern projective geometry, and the axiomatics of geometry, two of which appeared in *Mathematische Annalen*. His 1912 paper on the construction of geometry on the basis of only projective axioms was read by Voss at a meeting of the Bavarian Academy. In 1913 he published two notes in *Comptes Rendus* in connection with the use of iterative techniques for the solutions of algebraic equations. It is very possible that Müntz was the first to develop an iterative procedure for the determination of the smallest eigenvalue of a positive definite matrix. It certainly predates the more generally quoted result of R. von Mises of 1929 [8]. In 1914 he published an additional two papers on approximation theory. The first is a note on properties of Bernoulli polynomials published in *Comptes Rendus*. The other is the paper in which the Müntz Theorem appeared. This last work was written as a contribution to the *Festschrift* in honour of his teacher Hermann Schwarz's 70th birthday.

In this period reference is already made in Müntz's correspondence to serious problems in one of his eyes. Eye problems would continually plague Müntz throughout his life.

Boarding Schools and Martin Buber (1914 - 1919)

In early 1914, probably through socialist and feminist common friends, Müntz started a correspondence with the pedagogue Paul Geheeb, who ran a boarding school called the Odenwaldschule near Heppenheim in southern Hessen. Müntz moved to Geheeb's school in 1914 as a mathematics teacher, with the understanding that he would be able to devote a considerable amount of his time to his mathematical research. It was agreed that he would have at most three hours of teaching a day. This was to be the first time he taught very young children.

In a letter to Geheeb written by Mario Jona, who interviewed him for the position, there is the following passage:⁵ "He [Müntz] is perfectly aware of what he is worth and shows it, which face to face is not so unpleasant as in writing. As it was I imagined him from his letter to be much more terrible. He is short, pleasant and with a very serious appearance and sometimes a little clumsy in politeness, For him the most important thing is his scientific work. He is in a period of important scientific activity, but would like also to work in a school like ours if he also has time to work for himself."

⁵ Jona to Geheeb, March 10, 1914. Archive of the École d'Humanité.

Geheeb was a liberal humanist, pro-feminist and much opposed to anti-Semitism. He and his schools hold a special place in the history of progressive education in Germany. At one of his earlier schools, in Wickersdorf, he had established the first co-educational boarding school in Germany. His wife, Edith Cassirer, was a progressive young teacher, the daughter of the wealthy Berlin Jewish industrialist Max Cassirer. With his father-in-law's financial backing, Geheeb founded the Odenwaldschule in 1910. It was a large boarding school with modern or specially modernized buildings. Co-education, an emphasis on physical education and flexibility in the curriculum, were among its innovations. The new school was run with a fair amount of self-government. The teachers, and especially Geheeb, supposedly guided rather than led. The students were called *Kameraden*, "comrades," and the teachers *Mitarbeiter*, "co-workers." In 1914, the time we are talking about, there were 68 full-time students, many of whom were children of the liberal, affluent, German intelligentsia. The children of Thomas Mann and of other noted writers and artists were among the pupils and were not necessarily easy to handle. Much has been written about this school and Geheeb. The school survived both wars and exists today, but the Geheeb family left in 1934 and moved to Switzerland, when the influence of Nazi activists reached the school. There, he and his wife established a school of a related character: *École d'Humanité*.

According to some, Müntz included, life in Odenwaldschule seemed anarchic on occasions. Müntz and Geheeb parted ways in the summer of 1915. Nonetheless Müntz kept in touch with some of the school's faculty and remained on speaking terms with Geheeb.

Müntz then found a similar position at another school, Dürerschule, which does not exist today, in Hochwaldhausen also in Hessen. Müntz seems to have enjoyed his teaching, and was particularly interested in the teaching of mathematics and science to younger children, developing very definite opinions thereon. Another teacher who joined him at the Dürerschule was his friend and brother-in-law Herman Schmalenbach, married to his sister Sala. He later became a Professor of Philosophy at the University of Basel.

The war was also having its impact. Müntz was regarded as an "alien" with Hessian residency, but no German citizenship, and he was generally restricted in his travels. This had prevented a move to Heidelberg planned in 1915. In a letter dated August 1917 Müntz wrote that he had to stay in Hessen to avoid difficulties with the authorities. However as an "alien" he did not take part in the war. Although happy at the school, he was forced to leave as a consequence of being subjected to anti-Semitic remarks by the director.⁶

Many pupils, especially the Jews, also did not return to Dürerschule after the holiday. Müntz felt he had a responsibility for some of these children and decided to return as a private scholar to Heppenheim where he had friends, but not to Odenwaldschule. With his wife, he managed a small boarding house for students: a *Schülerpensionat*.

Despite his many obligations and worries Müntz still managed to carry on with his mathematics research. During this period he published five more papers, concerning problems in projective geometry, and the solution of algebraic equations and algebraic eigenvalue problems.

⁶ In an open meeting in 1917 the headmaster, G. H. Neuendorff, had called him a "little Polish Jew." See Butschli [9].

While still at the Odenwaldschule, Müntz had begun to correspond with Martin Buber, the enlightened and broad-minded philosopher, Zionist thinker and writer, who was then in Berlin. Buber was the spiritual leader of an entire generation of German-speaking Jewish intellectuals. He adhered to a form of tolerant “utopian socialism” he called “Hebrew humanism.”

In 1915 Müntz helped Buber find a house⁷ in the town of Heppenheim, where Buber and his family lived from 1916 until 1938. Buber then left for Palestine to take a Chair in Social Philosophy at the Hebrew University and subsequently had a distinguished career there. During the years of the First World War the two families kept in close contact and exchanged fairly intense and interesting correspondence.

In 1915 Buber founded and co-edited a journal called *Der Jude* [10] that for eight years was the most important organ of German-reading Jewish intellectuals. In a letter dated in November of that year, Buber invited Müntz to become one of his collaborators on this journal. He wrote “You are, of course, amongst the first whom I am asking to participate.”⁸ Müntz wrote 18 articles and notes for this journal, some quite lengthy, which he published under the pseudonym of Herman Glenn. It is an indication of the way in which Müntz’s contributions were valued that in the very first issue of *Der Jude*, the first article was signed by Buber, while the second was signed by Glenn (Müntz).

Göttingen and Berlin (1919 - 1929)

Around 1919 or 1920 Müntz seems to have had a nervous breakdown and was placed at a sanatorium in Gandersheim⁹ near Göttingen. We do not know exactly how long Müntz was in the sanatorium. The few letters available from this period are rather bleak. In a letter to Buber in September of 1923 Müntz recalled that he suffered a personal collapse in 1919-1920 and said he learnt from the experience to look at things from a distance, and in “this way they are no longer dangerous to me.”¹⁰

Toward the end of 1920 Müntz and his wife moved to his wife’s family farm in Poland to recuperate for some eight to ten months. Letters show that during this period the Müntzes, together with his wife’s brothers, considered emigrating to Palestine. But the economic situation there was far from encouraging and the idea was dropped. As he recuperated, Müntz took up mathematics again and from the farm traveled to Warsaw to attend seminars and to lecture on his research. This activity is reflected in a number of publications in the journal of the then recently founded *Polish Mathematical Society*.

In October of 1921 the Müntzes returned to Germany, moving into a boarding house in Göttingen. At that time the Schmalenbachs also lived in that city. It is not clear how the Müntzes supported themselves in Göttingen, but it was again probably through private teaching and with the help of their family. While in Göttingen Müntz did a considerable amount of mathematical research. During this time he wrote eleven papers, that were published between 1922 and 1927. They range over a number of topics including integral

⁷ Today called *The Martin Buber House*, it is home to the *International Council of Christians and Jews*.

⁸ Buber to Müntz, November 11, 1915, Buber Archives, JNUL, Jerusalem.

⁹ Now called Bad Gandersheim.

¹⁰ Müntz to Buber, September 18, 1923, Buber Archives, JNUL, Jerusalem.

equations, the n -body problem, summability, Plateau's problem, and quite a few papers on number theory,¹¹ possibly under the influence of Edmund Landau.

By this time Müntz seems to have made a name for himself within both mathematical and Jewish/Zionist circles. He was a member of the editorial board of the mathematics and physics section of the short-lived journal *Scripta Universitatis* founded by Immanuel Velikovsky from Jerusalem. The one and only issue of the mathematics and physics section was edited by Einstein and published in 1923. He also cooperated with Hertz, Kneser and Ostrowski in a German translation of some lectures of Levi-Civita [M21].

In April 1924 the Müntzes moved to Berlin, while maintaining scientific contacts in Göttingen. In Berlin they rented an apartment and seemed to live happily. Müntz also returned to writing on Jewish matters. He sent contributions to *Der Jude* some of which were excerpts from the third of his unpublished books on Jewish matters.

Müntz does not seem to have favored scientific collaboration, although he had at least one student in this period. Divsha Amirà (néé Itine) from Palestine was a geometer who officially obtained her doctorate from the University of Geneva in 1924. She collaborated with Müntz whilst residing in Göttingen. In 1925 she published a memoir [12] on a projective synthesis of Euclidean geometry. Müntz had attempted a construction of algebraic Euclidean geometry using what he called *Basisfiguren*. In her memoir Divsha extended Müntz's ideas to the general Euclidean plane considering, instead, sets of straight lines. She discussed elementary constructions, congruence axioms and the axiomatic construction of geometry. Divsha was generous in her remarks to Müntz and to his research. Although not her formal thesis supervisor, Müntz clearly was her mentor. Divsha's husband Benjamin, who also obtained his doctorate from the University of Geneva, was a student of Edmund Landau.

The first meeting of the board of governors of the new Hebrew University in Jerusalem took place in April of 1925. At that meeting it was decided to establish an institute devoted to research in pure mathematics, staffed by one professor and two assistants. The board of governors also authorized the President and Chancellor to offer Edmund Landau the professorship in pure mathematics. The involvement of Landau in the Hebrew University started well before the First World War, and lasted into the 1930's. He was the person who had the major say in the choice of who would be appointed to the mathematics institute. At a meeting of the board of governors in September 1925, Landau was asked to draw up plans for the establishment of a mathematical institute to be opened as soon as funding became available. At the suggestion of Landau, it was decided to appoint Benjamin Amirà as the first assistant.

In October of 1925 Müntz was in Berlin and was busy trying to find a position, either in or outside Germany. The creation of the Hebrew University undoubtedly interested him as a mathematician, as someone without proper employment and as a Zionist. Müntz saw himself as the professor and thus the director of this new mathematical institute. In a rather manipulative way he used his connections, particularly Schmalenbach's everlasting good disposition towards him. He asked his well-positioned brother-in-law to contact Buber, Landau and Courant on his behalf. Schmalenbach reported that Courant was emphatic in stating that he had no doubts as to Müntz's qualifications, which he exhibited in his papers and in his lectures at the meetings of the local mathematical society. However he indicated

¹¹ One of his results from this period is quoted in Titchmarsh [11].

that not having a Habilitation was a serious drawback in his case. The Jerusalem matter was resolved negatively in early November. In retrospect, it seems Müntz had misread, or been misled, concerning the entire situation. At this early stage in 1925 Landau probably was saving the professorship for himself. In any case the academic leadership of the Hebrew University was looking for an established star to take up the professorship. At the very least they were looking for someone with a Habilitation and also a chair somewhere else. They were actually looking for a person like Landau, who in fact moved to Jerusalem with his family for the initial academic year of 1927-28. However, for various reasons things did not work out and he returned to Göttingen the following year.

Throughout this period Müntz was constantly seeking an academic appointment, while at the same time attempting to obtain his Habilitation. In 1925 Voss and von Lindemann recommended him for a Habilitation in Giessen, but nothing came of it. That same year it appears he was recommended, by A. A. Fraenkel, for a position at the University of Cairo. Again he was unsuccessful.

As we said, according to Courant,¹² the fact that Müntz had not been given the Habilitation in Göttingen was not as a consequence of his lack of qualifications. There were other reasons. On the one hand there was Göttingen's hierarchy. As in the cases of Bernays, Hertz, and E. Noether, if he were not to be called by a university, Göttingen would feel morally obliged to provide for his maintenance. On the other hand there was the question of his origin, which Müntz did not try to hide. Müntz had heard essentially the same from Hilbert years earlier. To all this Müntz justifiably complained that he was in an impossible situation. If he was guaranteed a position then he would have no problems being given Habilitation, but without the Habilitation it was almost impossible to obtain a position. His case was not unique.

One source of income for Müntz we have identified is the *Jahrbuch über die Fortschritte der Mathematik* (FdM). This annual review, published from 1869 until the end of the Second World War, was in the format adopted by the *Zentralblatt für Mathematik*, and later shared by *Mathematical Reviews*, except that it appeared each year as a single volume. As a consequence of the First World War, the work on the annuals was severely backlogged and remained so for many years thereafter. A count of the reviews signed by Müntz, shows he wrote nearly 800 reviews for the FdM, mainly during the mid 1920's. In fact he was still registered among the journal's regular reviewers up to 1929 when he had already left Germany. Reviewers were paid 1 Reichsmark per review. The average salary at the time seems to have been about 120 Reichsmark per month.

The reviews by Müntz cover an extensive mathematical, as well as linguistic, area. Besides the languages he was brought up in, namely Polish, German, and Russian, Müntz reviewed papers in English, French, Italian, Dutch and Swedish. The topics frequented by him, besides function theory, and differential equations, were probability theory, fluid mechanics, the theory of electricity and magnetism, including its geophysical applications, numerical methods of calculation and the history of mathematics.

At the end of 1927 Müntz wrote to Buber that for several months he has been the professional scientific collaborator of Einstein, and added "This, of course, compensates me a great deal for what has been in Germany an almost impossible situation as the official

¹² As related by Schmalenbach.

professionals are more ‘official’ than ‘professional.’ ”¹³ He was probably alluding to the fact that he had been unable to obtain a Habilitation. It is not totally clear when Müntz started to work with Einstein and for how long this collaboration continued. From his wife’s correspondence it seems he met Einstein socially in January 1927. For much of the time that Müntz worked with/for Einstein, another subsequently well-known mathematician, Cornelius Lanczos, also worked with Einstein. Both seem to have been supported by grants from a fund supporting promising “young” scientists.¹⁴ Müntz published no joint papers with Einstein. But Einstein’s archive has an extensive collection of correspondence between Müntz and Einstein on a range of mathematical ideas. Moreover Müntz, as well as Lanczos, are mentioned and thanked in two of Einstein’s papers on distant parallelism.

In describing to his sister his work under Einstein in September of 1927, Müntz indicated that it was “running ‘normally’ and for reasons of convenience I have ‘submitted’ myself; for in these fields it is he who is the extraordinary master while I am only the ‘technical’ assistant. Nevertheless I am very happy to be working with him.” On more than one occasion, however, Einstein politely expressed reservations regarding Müntz’s work. Einstein sometimes indicated that he did not believe there were sufficient reasons for Müntz’s assumptions, or he did not regard Müntz’s reasoning as being justified, or he did not think that Müntz’s arguments made “any obvious experimental-physical sense.”

Toward the end of 1928 Müntz was again considering the possibility of taking a chair outside Germany. However, he purposely kept these discussions from many of his close friends and colleagues, including Lichtenstein and Einstein,¹⁵ which suggests that he still expected that they might be able to help him find a job within Germany.

Leningrad (1929 - 1937)

In May of 1929 Müntz finally obtained an academic appointment, something he had yearned for for many years. He was invited to fill the position of Professor of Mathematics and Head of the Chair of Differential Equations at the Leningrad State University. In Leningrad Müntz was also put in a group of “exceptional scientists,” and given a “personal salary.” From 1933 he is listed as Head of the Chair of Differential and Integral Equations.

In a letter written a few years later, Müntz stated that in 1927 he was also offered the Lobatschevsky Chair in Kazan, and that during the technical period of waiting was offered the Chair for Higher Analysis in Leningrad. According to Müntz he “exchanged” the chair in Kazan with that in Leningrad (initially offered to Bernstein), while his friend N. G. Chebotarev took the Kazan Chair. We have found no direct documentation to support this claim, but the fact is that Chebotarev became professor at Kazan University in 1928 after having been offered posts at both Kazan and Leningrad.

G. G. Lorentz, who was an undergraduate at the time, recalled [13] that in 1930, shortly after his arrival in Leningrad, Müntz was called upon to present a lecture sponsored by the *Leningrad Physical and Mathematical Society* on the so-called crisis of the exact sciences. The subject matter was the foundational debate in mathematics, and Hilbert’s attack on the intuitionism of Brouwer and Weyl. Müntz was an ideal candidate to deliver the lecture

¹³ Müntz to Buber, October 30, 1927, Buber Archives, JNUL, Jerusalem.

¹⁴ The *Notgemeinschaft Deutscher Wissenschaftler*.

¹⁵“Einstein (as well as Lichtenstein) as well as the rest of the insider world shall not learn anything of this.” Müntz to Schmalenbach, December 5, 1928.

because of his research background on the foundations of mathematics; and having recently arrived from Germany, he was perceived as the carrier of the latest advances on this controversy.

The lecture was well attended. Of course Müntz stated that the crisis was only in the “foundations” and did not in any way affect the work of most mathematicians. However, because of an underlying power struggle between N. M. Günter, V. I. Smirnov and Ya. V. Uspensky, on the Society’s traditional side, and L. A. Leifert and E. S. Rabinovich, of an alternative young Communist league, the meeting turned rowdy and undisciplined. The *Leningrad Physical and Mathematical Society* subsequently ceased to exist in its previous form, being amalgamated into a new organization under Rabinovich.

The row does not seem to have affected Müntz’s subsequent career. From 1931 Müntz was also in charge of mathematical analysis at the Scientific and Research Institute in Mathematics and Mechanics (*Nauchna Issledovatelskii Institut Matematiski i Mechanika*, or NIIMM) at Leningrad State University. Furthermore, in 1932, Müntz’s position in Russia seems to have been quite firm, since he was given the singular distinction of being sent to the *International Congress of Mathematicians* in Zurich as one of four of the Soviet Union’s official delegates. The other three were Chebotarev, representing Kazan State University, who gave a plenary lecture on Galois Theory (on the occasion of the centenary of the death of Galois), the famous topologist P. S. Aleksandrov from Moscow State University, who talked about Dimension Theory, and É. Ya. Kolman, a mathematician and a member of the *Communist Academy*, in Moscow. The Academy was an institution created in 1918 which had been given the task of developing Marxist views in the fields of philosophy and science. Kolman, the ideologist in this delegation, gave two talks, the first about quaternions and the second about the foundations of differential calculus in the works of Karl Marx. Müntz, representing NIIMM at Leningrad State University, read a paper on boundary value problems in Mathematical Physics.

While Müntz had been unable to obtain his Habilitation in Germany, he was far more successful in Russia. In 1935, at the recommendation of Leningrad State University, VAK, the committee that gave these higher (or second) doctorates in Russia, awarded Müntz a higher degree without requiring the submission of a written thesis. Müntz would later write that he had been awarded an honorary doctorate, and that could be one possible interpretation of this degree. All the above testifies to the fact that without doubt Müntz held a senior position in Leningrad State University and had the respect of his colleagues. He had fulfilled his ambition.

Müntz seems to have been active administratively, pedagogically and mathematically. In a later letter to Einstein he wrote about working on a uniform theory of the solutions of non-stationary boundary value problems in homogeneous and non-homogeneous spaces. However he also talked about the heavy teaching and administrative load, and the unfortunate state of his eyes that hindered him greatly. In 1934 he published a textbook on *Integral Equations* [M32] which is still sometimes referenced, and in 1935 he edited a Russian edition of Liapounoff’s important monograph on *General Problems of Stability of Motion* [M35]. The fact that Müntz was given this task, of historical as well as scientific importance, is another indication of the high regard in which he was held at the time. He also wrote some half dozen research papers, mainly on boundary-value problems, integral equations and on topics of Mathematical Physics. Furthermore, he was asked to write a review of his own work for the *Second All-Union Mathematical Congress* held in Leningrad

in 1934. The latter was a definite honour, awarded at a time when he began to recover from further eye problems.

While in the Soviet Union Müntz kept a low, neutral, profile vis-à-vis internal politics. Although he kept his German citizenship, obtained in 1919, at some stage he was given a “former foreigner” status within the Soviet Union. He also seems to have traveled abroad widely in the company of his wife, visiting Finland, Germany, Switzerland and Poland. Generally visits were related to vacations or had to do with mathematics research or meetings, but sometimes they were motivated by his and his wife’s health.

While visiting Berlin on vacations, by March 1930 Müntz¹⁶ was again working under Einstein. His work related to a question “of compatibility of partial differential equations” which Einstein indicated had been solved by Cartan, in a wonderful fashion, but had not yet been published. He indicated to Müntz “You will take pleasure from it.”

Magda had suffered a cerebral thrombosis in 1934. Müntz himself, as has been mentioned, suffered from severe problems in one eye and around 1934, at the beginning of term, he suffered damage in the retina of the other eye. This kept him away from his academic duties for several months. As he began to recover, his department provided a secretary to help him with his research and when in a position to teach, his students helped him by writing on the blackboard before each lecture the formulae he needed.

F. I. Ivanov, who was a graduate student in the 1930’s, recalls that Müntz conducted a seminar in Mathematical Physics. Ivanov remembers Müntz as being very actively occupied with science, but both accessible and sociable. He writes that Müntz was stout, had light-grey hair and very strong glasses because of his poor eyesight. For this reason Müntz’s wife would bring him to the seminar.

According to Müntz he also helped, sometimes directly and sometimes indirectly, mathematicians from Central Europe obtain positions in the Soviet Union. He said that the appointments of Cohn-Vossen, a former collaborator of Hilbert who died of pneumonia shortly after arriving in Moscow, and of the number theorist Walfisz, who went to Tbilissi, were the result of his suggestions. With others he helped both Plessner and Bergman obtain positions.

In October 1937 Müntz was expelled from the Soviet Union without, according to him, any apparent reason. However this was part of a wide movement sweeping the country, which turned the tide against foreigners, including teachers and engineers. It even affected those, like Müntz, who had helped develop different areas of activity in the Soviet Union. The Müntzes were given a few weeks to leave.

Sweden (1937 - 1956)

Leaving the Soviet Union was a shock to Müntz who was then 53 years old. He had lost the position he had worked most of his life to obtain. The Müntzes were permitted to take their personal possessions with them, but no financial recompense was offered for his eight years as a professor at Leningrad State University.

¹⁶ Letter to his sister Sala, March 28, 1930.

Travelling long distances was risky because of his wife's thrombosis. The Müntzes left Russia and first went to Tallinn in Estonia. According to Müntz, the Mathematics-Mechanics Faculty of the Technical University in Tallinn offered him a visiting professorship for the spring semester. But since neither German nor Russian were then acceptable teaching languages - ministry regulations required lectures to be given in Estonian - this was not a possibility. In February of 1938, the Müntzes moved to Sweden where they later requested political asylum.

Immediately after being expelled from Russia, Müntz asked for help in obtaining an academic position from a number of colleagues and former teachers. He also asked family and personal friends for financial help. Müntz's file at the *Society for the Protection of Science and Learning* and files on him in other scientific refugee organizations indicate that he contacted, among others, Harald Bohr, Einstein, Landau, Levi-Civita, Volterra, Weyl and Courant.

Finding an academic position for a scientist of Müntz's age was not easy. Technically he was not a refugee from Hitler's Germany, since he had left in 1929. This put him outside the purview of relief organizations such as the *Notgemeinschaft Deutscher Wissenschaftler im Ausland*. Furthermore, he was already in Sweden, a relatively safe place. Because of earlier emigration from Germany, competition for academic positions in Europe and in the United States was fierce and involved a large number of parameters to overcome the resistance and anti-foreign feeling, often as intense as the generosity of those many prepared to help. Seniority in the field, age of the candidate, area of research, even personality, without leaving aside the strength of his personal network of scientific contacts, were among these parameters.

While Müntz was referred to as a "mathematician of the highest rank," in the dossiers of agencies dealing with displaced scientists, he was not considered a star. There were other drawbacks in Müntz's prospects for employment. Crucially, Einstein had concerns regarding Müntz's personality, dating from their association in the late 1920's, which we have briefly outlined. In a letter to a colleague, Einstein explained his reservations in terms of what he regarded as Müntz's inability to submit his ideas to a proper level of critical analysis and his previous mental ill-health. Einstein thought that, in a period of general distress, he should reserve his influence to more clear-cut cases. An imbalance in his personality, probably associated with his nervous breakdown of 20 years earlier, was now a serious drawback in Müntz's prospects for employment.

Müntz resented Einstein's attitude and especially Einstein's suggestion (in 1938!) that Müntz should look for work in his native Poland. Müntz indicated that he was now in exile from Poland, Germany and the Soviet Union, and in Germany he had not been forgiven for his former cooperation with Einstein. The tremendous competitiveness for jobs in America at the time may not have been entirely clear to Müntz, as the job-description of his aspirations suggest. In a possibly lost letter, Einstein may have pointed to such difficulties. Only Müntz's reply to this letter is available. Müntz responded that it was painful and unjust after so many years "to conclude that my present fate is to be judged only by the statistics of supply and demand." Even if Müntz was right, this last matter was clearly not in Einstein's hands.

In Sweden, as previously in Germany, Müntz had less success in penetrating official academic circles than in the Soviet Union. However, from the first days of his arrival there he

had the support of Professor Göran Liljestrand, chairman of the funding committee for exiled intellectuals, who helped him financially in 1939 and 1940. A number of distinguished Swedish academics also offered their help and friendship, among them Professors Folke K.-G. Odqvist, a mechanics specialist, Hugo Valentin, a physicist, Marcus Ehrenpreis, a pediatrician surgeon, and David Katz, a psychologist.

Initially, in 1940-42, he received research grants from the *Karolinska Institutet* for work on mathematical problems related to haemodynamics, the study of the dynamics of blood flow, whose problems involved the solution of complex non-linear partial differential equations. Research on this subject was carried out at the Maria Hospital, in Stockholm, in collaboration with a young medical doctor, Dr. A. Apéria. Müntz published a note on haemodynamics in *Comptes Rendus*, submitted by Hadamard, that appeared in the February 1939 issue. In 1942 Apéria died and this research came to an abrupt end.

Although he was on good terms with some leading Swedish scientists and intellectuals, Müntz was not able to forge a working contact with the small but active Swedish mathematical community of the time. Nevertheless, for some years he remained interested in various mathematical problems. In correspondence with Einstein and others he indicated he was interested in problems of integral equations, turbulence, knot theory, actuarial mathematics and, of course, haemodynamics. However, possibly due to the severity of his circumstances, no subsequent scientific papers of Müntz have come to light from this last period.

While in Stockholm Müntz returned to private teaching, and the couple moved to an apartment in Solna, a pleasant district of Stockholm. They had a telephone in their name, which suggests that their financial circumstances had improved. It seems that they did have some outside financial resources. Müntz later received a small pension from the *Warburgfonden*, a foundation controlled by the “Mosaic” community in Sweden.

His wife Magdalena died of hemiplegia on January 19, 1949. Müntz became a Swedish citizen in 1953 and died on April 17, 1956, at the age of 71. He was blind for the last few years of his life. But for an obituary in *Svenska Dagbladet*, the leading Swedish newspaper, written by Odqvist, his death passed almost unnoticed to the mathematical community of Sweden and the rest of the world. In this obituary Professor Odqvist summarized the last years of Müntz’s life in a short but poignant paragraph: “Herman Müntz is dead. In spite of the fact that he lived in Sweden for 18 years, the last five years¹⁷ as a Swedish citizen, there are probably not many Swedes outside his nearest circle of acquaintances, that knew that we had among us a mathematician of international fame who was thrown up on our calm shore by the storms of the times, his life saved but with his scientific activities broken.” He ended the obituary with these words: “Herman Müntz lived in an exceptionally harmonious marriage and his wife Magda meant much to him, not in the least in order to keep his floating spirit down to earth. After her death in 1949 he only seldom saw his friends and he went every day to her grave in the Jewish cemetery with fresh flowers as long as he could. Now he is gone. Let this be a modest flower of memory from his Swedish friends. Let his memory be blessed.”

¹⁷ Odqvist is wrong in this point, Müntz was granted Swedish citizenship in 1953. Riksarkivet, Stockholm.

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Herman Müntz: List of Mathematical Publications¹⁸

- [M1] Zum Randwertproblem der partiellen Differentialgleichung der Minimalflächen, *J. Reine Angew. Math.*, **139** (1911), 52-79.
- [M2] Aufbau der gesamten Geometrie auf Grund der projektiven Axiome allein, *Münchener Sitz.*, (1912), 223-260.
- [M3] Das Euklidische Parallelenproblem, *Math. Ann.*, **73** (1913), 241-244.
- [M4] Das Archimedische Prinzip und der Pascalsche Satz, *Math. Ann.*, **74** (1913), 301-308.
- [M5] Solution directe de l'équation séculaire et de quelques problèmes analogues transcendants, *C. R. Acad. Sci. Paris*, **156** (1913), 43-46.
- [M6] Sur la solution des équations séculaires et des équations intégrales, *C. R. Acad. Sci. Paris*, **156** (1913), 860-862.
- [M7] Über den Approximationssatz von Weierstrass, in *H. A. Schwarz-Festschrift*, Berlin, 1914, 303-312.
- [M8] Sur une propriété des polynômes de Bernoulli, *C. R. Acad. Sci. Paris*, **158** (1914), 1864-1866.
- [M9] Ein nichtreduzierbares Axiomensystem der Geometrie, *Jber. Deutsch Math. Verein*, **23** (1914), 54-80.
- [M10] Approximation willkürlicher Funktionen durch Wurzeln, *Archiv Math. Physik*, **24** (1916), 310-316.
- [M11] Zur expliziten Bestimmung der Hauptachsen quadratischer Formen und der Eigenfunktionen symmetrischer Kerne, *Gött. Nachr.*, (1917), 136-140.
- [M12] On projective analytical geometry (in Polish and German), *Prac. Mat.-Fiz.*, **28** (1917), 87-100.
- [M13] The problem of principal axes for quadratic forms and symmetric integral equations (in Polish and German), *Prac. Mat.-Fiz.*, **29** (1918), 109-177.
- [M14] A general theory for the direct solution of equations (in Polish), *Prac. Mat.-Fiz.*, **30** (1919), 95-119.

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- [M16] Allgemeine independente Auflösung der Integralgleichungen erster Art, *Math. Ann.*, **87**, 1-3 (1922), 139-149.
- [M17] Beziehungen der Riemannschen ζ -Funktion zu willkürlichen reellen Funktionen, *Mat. Tidsskrift B*, (1922), 39-47.
- [M18] Absolute Approximation und Dirichletsches Prinzip, *Gött. Nachr.*, **2** (1922), 121-124.
- [M19] Allgemeine Begründung der Theorie der höheren ζ -Funktionen, *Abhdl. des Sem. Hamburg*, **3** (1923), 1-11.
- [M20] Der Summensatz von Cauchy in beliebigen algebraischen Zahlkörpern und die Diskriminante derselben, *Math. Ann.*, **90**, 1-3 (1923), 279-291.
- [M21] *Fragen der klassischen und relativistischen Mechanik. Vier Vorträge gehalten in Spanien im Januar 1921*, by T. Levi-Civita; authorized translation by P. Hertz, H. Kneser, Ch. H. Müntz, and A. Ostrowski, pp. VI + 110, J. Springer, Berlin, 1924.
- [M22] Umkehrung bestimmter Integrale und absolute Approximation, *Math. Z.*, **21** (1924), 96-110.
- [M23] Über den Gebrauch willkürlicher Funktionen in der analytischen Zahlentheorie, *Sitzungsberichte der Berliner Math. Gesellschaft*, **24**, 2 (1925), 81-93.
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- [M25] Zur Gittertheorie n -dimensionaler Ellipsoide, *Math. Z.*, **25** (1926), 150-165.
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- [M27] Über die Potenzsummen einer Entwicklung nach Hermiteschen Polynomen, *Math. Z.*, **31**, 2-3 (1929), 350-355.
- [M28] Sur la résolution du problème dynamique de l'élasticité, *C. R. Acad. Sci. Paris*, **194** (1932), 1456-1459.
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[M34] Functional Methods for Boundary Value Problems (in Russian), *Works of the 2nd All-Union Mathematical Congress, Leningrad*, Leningrad-Moscow, **1** (1935), 318-337.

[M35] *General problems of stability of motion*, by A. Liapounoff, (in Russian), Ch. H. Müntz, ed., Leningrad-Moscow, 1935.

[M36] Zur Theorie der Randwertaufgaben bei hyperbolischen Gleichungen, *Prace Mat.-Fiz.*, (Gedenkschrift für L. Lichtenstein), **43** (1936), 289-305.

[M37] Les lois fondamentales de l'hémodynamique, *C. R. Acad. Sci. Paris*, **280** (1939), 600-602.

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