MR2584277 (2010k:15065) 15B48

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★**Totally positive matrices.**

Cambridge Tracts in Mathematics, 181.

Cambridge University Press, Cambridge, 2010. *xii*+182 *pp*. $68.00. *ISBN* 978-0-521-19408-2

This monograph is devoted to (finite) totally positive (TP) and strictly totally positive (STP) matrices. An m-by-n matrix is TP (STP) if all its minors are nonnegative (positive). An oscillation matrix is a TP matrix such that some power is STP. By and large, analysts initiated the study of TP matrices and most of the texts on the subject have had an emphasis on analysis, as opposed to matrix theory. However, in 1987 Ando wrote a short monograph devoted to TP matrices. This subsequent monograph is devoted to research in this area over the past 20 years; it updates and expands Ando's monograph.

The book is divided into six chapters. The first chapter introduces some of the notation, basic definitions, and classical facts and formulas. The second chapter establishes determinantal criteria for a matrix to be TP (STP). In particular, Fekete's Lemma is proved. In the third chapter two fundamental results on variation diminishing properties are proved and an application is given. In the fourth chapter various examples of TP (STP) are presented as well as subclasses of TP (STP) matrices that are closed under Hadamard (entry-wise) multiplication. Chapter five is devoted to the study of spectral properties of TP (STP) matrices. It is shown that an STP matrix has positive, simple eigenvalues with the associated eigenvectors possessing an intricate structure. This is not the case for TP matrices, but oscillation matrices are shown to have the same spectral properties as those of STP matrices. The Gantmacher-Kreĭn Theorem is proved. Interlacing properties of eigenvalues of principal submatrices of TP (STP) are derived. In the sixth and final chapter factorizations of TP (STP) matrices are studied in detail.

This comprehensive monograph will be of interest to any matrix theorist and to analysts working in total positivity. The central properties of TP matrices are presented with full proofs. An excellent bibliography and history of the topic are provided, with a tribute to I. J. Schoenberg, M. G. Kreĭn, F. R. Gantmacher, and S. Karlin—the mathematicians who have made the most significant contributions to the topic.

Reviewed by *Ronald L. Smith*