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Pinkus, Allan (IL-TECH)
Density in approximation theory. (English summary)
Surv. Approx. Theory **1** (2005), 1–45 (*electronic*).

The paper under review is a survey on density theorems in approximation theory. It presents the history of the Weierstrass theorem and its offshoots. The author has thoroughly researched early and more recent developments, and while the paper is easily readable for a novice, the expert will also find some interesting tidbits of information.

About 25 years after Weierstrass showed in 1861 how "bad" a continuous function can be, he gave the now famous Weierstrass approximation theorem showing that any continuous function is "nearly good"; that is, any $f \in C[a,b]$ can be approximated by polynomials. A. Pinkus describes the original view of Weierstrass on his theorem. He outlines the Lebesgue proof that reduced the problem to the approximation of $|x|$. The other proof commonly used today, i.e. the Bohman-Korovkin technique, is also given. Pinkus also discusses the functional analytic approach dealt with in the first few decades of the last century by several distinguished mathematicians. In addition, he gives the Stone generalization of the Weierstrass theorem, described in most intermediate texts (and up) as the Stone-Weierstrass theorem. Furthermore, he outlines in detail the Müntz and Bernstein density results. The Müntz theorem yields conditions on λ_i for $\operatorname{span}\{x^{\lambda_i}\}$ to be dense. The problem posed by Bernstein is: For what weights $w(x)$ are polynomials dense in the norm $\|fw\|_{C(\mathbb{R})}$ under the condition $\lim_{|x|\to\infty} w(x)f(x)=0$? This problem led to a necessary and sufficient condition by Mergelian. The most common and interesting weights for which the density of polynomials are discussed on $\mathbb{R}$ are the so-called Freud weights. For the multivariate situation, the reduction of the problem to a lower number of variables and the density of ridge functions are discussed.

In all, this is an interesting and recommended survey of density theorems in spaces of continuous functions by a restricted set of "simpler" continuous functions. Because this is already quite a long survey, the problems of density in other spaces is not dealt with. The treatment here is of density alone; that is, the results described are of a qualitative, not quantitative, nature.

Reviewed by *Z. Ditzian*