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Necessary conditions for uniqueness in $L^1$-approximation.


The model theorem for this paper is the classical theorem identifying the finite-dimensional Chebyshev subspaces of $C(X)$ with the Haar subspaces. It is also classically known that the Haar subspaces of $C(X)$ are an example of Chebyshev subspaces of $C_1(X)$, the incomplete normed space composed of $C(X)$ normed with the $L_1$ norm. However the family of Chebyshev subspaces of $C_1(X)$ has eluded characterization. Perhaps the reason is that the Chebyshev subspaces of $C(X)$ are characterized solely by properties of $Z(h)$, the zero sets of its members $h$. Hence $H$ is a Chebyshev subspace of $C(X)$ if and only if $Hw = \{hw: h \in H\}$ is Chebyshev for all continuous $w > 0$. This statement is not true for Chebyshev subspaces of $C_1(X)$.

An interesting approach then is to characterize the subspaces of $C_1(X)$ that are Chebyshev for all such weight functions. Pinkus did this for $X = [a,b]$ [same journal 48 (1986), no. 2, 226–250; MR 88a:41014]. The characterizations require that for each $h$ in $H$ the number of components created by deleting $Z(h)$ from $X$ does not exceed the dimension of the space of functions in $H$ that vanish on $Z(h)$. The present work extends these results to compact sets $X$ contained in $\mathbb{R}^n$ which are closures of their interiors. Other significant contributions to this area are in papers by M. Sommer [Numer. Funct. Anal. Optim. 9 (1987), no. 1-2, 131–146; MR 88e:41062] and A. Kroó [J. Approx. Theory 51 (1987), no. 2, 98–111; MR 89b:41036].

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