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Continuous selections for the metric projection on $C_1$.

Let $K$ be the closure of a bounded open set in $\mathbb{R}^n$, and denote by $C_1$ the normed space of continuous real-valued functions on $K$ with the usual $L_1$-norm. Let $U$ be a finite-dimensional subspace of $C_1$. Does there exist an $L_1$-continuous map $A: C_1 \to U$ such that $\|f - Af\| = \text{dist}(f, U)$ for each $f \in C_1$? The author proves that if such a map exists then $U$ is a “unicity” space; i.e., each $f$ has exactly one best approximation in $U$. (All metric notions are relative to the $L_1$-norm.) If $K$ is connected, the same conclusion can be drawn if $A$ is only $L_\infty$-continuous. The proofs are lengthy and ingenious.

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