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The inverse of a totally positive bi-infinite band matrix.

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Inverses of infinite sign regular matrices.

A few years ago Ch. Micchelli made a conjecture in the framework of cardinal spline interpolation which in terms of matrices can be phrased: “A bi-finite banded totally positive matrix $A$ is boundedly invertible if and only if the linear system $Ax = ((-1)^i)$ has a (unique) bounded solution.” To be precise, it was formulated without the uniqueness condition given in parenthesis. In this paper it is shown that the conjecture is true and that the supplement on uniqueness is necessary. To this end the “algebraic” kernels of strictly $m$-banded matrices are studied, and in this connection finite sections of $m$-banded matrices are investigated. The variation-diminishing properties of totally positive matrices imply that any vector in the kernel carries sign changes, i.e., $x_i, x_{i+1} < 0$ with at most $m$ exceptions. This gives rise to inclusions for the solution of the system $Ax = ((-1)^i)$. Furthermore the matrix $A$ is shown to have a main diagonal, i.e., the inverse of $A$ is the bounded pointwise limit of inverses of finite sections of $A$, principal with respect to a particular diagonal.


In the second paper the problem is treated without the assumption that the matrices are $m$-banded. (The other generalization, i.e., the extension from total positivity to sign regularity, is less spectacular.) Therefore another assumption is needed to prove the conjecture; here it is the assumption that $A$ carries $l_\infty$ to $l_\infty$. Any solution of $Ax = ((-1)^i)$ or, more generally, each uniformly alternating vector $Ax$ gives rise to a norm on the sequence space such that $A^{-1}$ is bounded.

A note added in proof reports on recent progress: R. Q. Jia proved that an invertible totally positive matrix carrying $l_\infty$ to $l_\infty$ must carry $c_0$ to $c_0$. This opened the way to show that such a matrix has one and only one main diagonal [de Boor, Jia and Pinkus, Linear Algebra
Appl. 47 (1982), 41–55. With this result the circle is closed.

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