Pinkus, Allan

Matrices and $n$-widths.


The paper is concerned mainly with the calculation of and relations between four types of $n$-width of the set \{\(Dx: \|x\|_q \leq 1\}\), where \(D\) is a real diagonal matrix with positive diagonal elements. These widths are those associated with the names of Bernšteǐn, Gel’fand and Kolmogorov as well as the anonymous linear $n$-width or approximation number. (The last is the minimum of \(\max \|Dx - Px\|_q\) taken over all matrices \(P\) of rank \(\leq n\) with \(\|x\|_p \leq 1\).) The $n$-width arose in approximation theory, and the author succeeds in showing that there are interesting and challenging problems in a finite-dimensional context. For example, he states that the problem of approximating even the identity matrix by matrices of fixed rank is unsolved in certain norms. The methods of analysis range from manipulations with inequalities to the use of Borsuk’s theorem on antipodal maps. It is not easy to summarize this paper mathematically; however, it will be of interest to workers in the union of approximation theory and matrix theory.

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