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Some problems in the approximation of functions of two variables and $n$-widths of integral operators.
For a nondegenerate totally positive kernel $K(x, y)$, the authors obtain a best choice of functions $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ for the problem $\min \{(\int_0^1 (\int_0^1 |K(x, y) - \sum_{i=1}^n u_i(x)v_i(y)|dy)dx)^{1/p}; u_i \in L^p[0,1], v_i \in L^1[0,1], i = 1, 2, \ldots, n, p \in [1, \infty]\}$. The authors also determine the $n$-widths (in the sense of Kolmogorov and of Gelfand) and identify optimal subspaces for the set $\mathcal{K}_{r,p} = \{f: f(x) = \sum_{i=1}^r a_i k_i(x) + \int_0^1 K(x, y)h(y)dy, ||h||_p \leq 1\}$, as a subset of $L^q[0,1]$, with either $p = \infty$ and $q \in [1, \infty]$, or $p \in [1, \infty]$ and $q = 1$, where $k_1(x), k_2(x), \ldots, k_r(x)$ and $K(x, y)$ satisfy certain restrictions.
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