Some extremal properties of perfect splines and the pointwise Landau problem on the finite interval.

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Let $W^{(n)}(\sigma) = \{ f: f^{(n-1)} \text{ absolutely continuous on } [0,1], \|f\|_\infty \leq 1, \|f^{(n)}\|_\infty \leq \sigma \}$. The Landau problem on $[0,1]$ for $W^{(n)}(\sigma)$ (as yet unsolved) asks for the value and the extremal function in the problem $\max \{ \|f^{(k)}\|_\infty : f \in W^{(n)}(\sigma) \}$, for $k = 1, \cdots, n-1$. This problem on $(-\infty, \infty)$ has been solved by Kolmogorov and on $[0, \infty]$ by Cavaretta and Schoenberg. The author essentially considers a pointwise version of the above, viz. $\max \{ |f^{(k)}(\xi)| : f \in W^{(n)}(\sigma) \}$ for fixed $\xi \in [0,1]$ and $k \in \{1, \cdots, n-1\}$. For each fixed $\sigma$, the author determines a one-parameter family of perfect splines of degree $n$ for which there exists a solution to this problem for each $\xi \in [0,1]$ and $k \in \{1, \cdots, n-1\}$ (i.e., the family depends on $\sigma$, but not on $\xi$ or $k$). Furthermore, each element of this one-parameter set is essential in the sense that for each $k$ it attains the above maximum for some $\xi$. The method of proof utilizes extremal properties of perfect splines, fixed point arguments, careful zero counting, and exact numerical differentiation formulae.

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