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Strictly positive definite functions on a real inner product space. (English. English summary)

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Let B be a subset of a real inner product space H. Then $f\colon \mathbf{R} \to \mathbf{R}$ is said to be (strictly) positive definite on B if all the matrices $[f(\langle x_j, x_j\rangle)]_{i,j=1}^n$ are (strictly) positive definite for all possible choices of the x_j from B and for all n. It is known that if f is of the form $f(t) = \sum_{k=0}^\infty a_k t^k$ converging for all t with all $a_k \geq 0$, then it is positive definite on any inner product space, but the converse is not true in general. The main result of this paper is a precise characterization of strictly positive definite functions: if f is given by a power series as before and $K = \{k\colon a_k > 0\}$, then f is strictly positive definite on $B \subset H$ if and only if K contains 0 and infinitely many even and infinitely many odd integers. Here H can be any real inner product space but the subset B should contain a ball with positive radius $c > 0$ centered in the origin of H. *A. Bultheel* (B-KUL-C)