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Nonlinear eigenvalue problems for a class of ordinary differential equations. (English. English summary)


In the paper the authors investigate eigenvalue problems associated with the homogeneous differential equation

\[(a_{n-1}(x)(a_1(x)(a_0(x)u^{p_0}u')^{p_1}u')^{p_2}...u')^{p_{n-1}})' = \lambda b(x)u^{r},\]

where the odd power function is defined by \(u^{p_*} := |u|^{p} \text{sgn } u\), \(p_i\) are positive real numbers satisfying \(p_0p_1...p_{n-1} = r\), \(a_i, b\) are positive functions and \(a_i \in C^{(n-i)}, b \in C\). The assumption on the numbers \(p_0, ..., p_{n-1}\) implies that the solution space of (*) is homogeneous and hence this equation is a higher-order extension of the so-called half-linear second-order differential equation \((a(x)(y')^{p_0})' = b(x)y^{r}\) which has attracted considerable attention in recent years.

Denote \(N_0[u] = a_0(x)u^{p_0}, N_i[u] = a_i(x)((N_{i-1}[u])')^{p_i}, i = 1, ..., n,\) with \(a_n \equiv 1, p_n = 1\). Equation (*) is considered with two-point boundary conditions of the form

\[(**) \quad N_i[u](a) = 0, \ i = i_1, ..., i_k, \quad N_j[u](b) = 0, \ j = j_1, ..., j_{n-k}\]

for some \(k \in \{1, ..., n-1\}\). Under some technical assumptions on the boundary conditions it is shown that (**) possesses an infinite sequence of real eigenvalues satisfying \(0 < (-1)^{n-k} \lambda_1 < (-1)^{n-k} \lambda_2 < \ldots, \lambda_i \to \infty\). The principal method used in the proof of this statement is a “connection” of the BVP (*)& **) to a certain linear BVP for which the above-mentioned eigenvalue result is known. This linear result is then carried over to (*) , (**) via this connection.

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