Some remarks on zero-increasing transformations. (English. English summary)


For any polynomial $p$, let $Z_I(p)$ denote the number of zeros of $p(x)$ in the real interval $I$. Furthermore, let $T$ be any linear operator which maps polynomials of degree at most $n$ onto polynomials of degree at most $n$.

The problem discussed here is to attempt to characterize the set of all such operators for which

\[ Z_I(p) \leq Z_I(Tp) \]

for all real-valued polynomials $p$ of degree at most $n$. Such operators are termed zero-increasing.

This problem has a long and distinguished history, dating back to Hermite and Poulain.

Presented here is a good history of the problem, and of the current state of knowledge, including the incomplete characterization for the case when the interval $I$ is the whole real line.

{For the entire collection see 2003c:41001}