Pinkus, Allan (IL-TECH)

Uniqueness in vector-valued approximation. (English. English summary)


The author studies best approximation by elements of finite-dimensional subspaces in the space of all $\mathbb{R}^m$-valued functions $f: x \in D \rightarrow f(x) = (f_1(x), \ldots, f_m(x)) \in \mathbb{R}^m$, where $D$ is some set, in the norms $\|f\|_{A(p,q)} = \left( \sum_{i=1}^{m} (\int_D |f_i(x)|^q \nu(x))^{p/q} \right)^{1/p}$ and $\|f\|_{B(p,q)} = \left( \int_D (\sum_{i=1}^{m} |f_i(x)|^q \nu(x))^{p/q} \right)^{1/p}$, where $p, q \in [1, \infty]$, with the usual conventions for $p = \infty$ and/or $q = \infty$. Some known results are surveyed and some new results are given on the characterization of Chebyshev subspaces and on the best approximants. Simultaneous approximation and tensor product approximation are also considered, as special cases. The bibliography contains 36 items.

{Reviewer’s remark: Many authors use the term “vector-valued approximation” for best approximation by vector-valued norms, in the sense of Pareto optimum.} 

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