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**On $L^1$-approximation.**


The model for $L_1$ approximation is the immensely beautiful and successful theory of Chebyshev approximation of continuous functions. The $L_1$ theory, although equally important, seems to be inherently more technical and despite the fact that it is a classical topic in analysis, many of its basic properties have only recently been discovered. This monograph is an excellent compendium of the current best known qualitative results on best approximations from finite-dimensional subspaces of $L_1$ spaces.

The unifying inquiries of the monograph are the characterization of best approximations, uniqueness of approximation, the dimension of the set of approximates, the size of the set of functions having unique best approximations, and the existence of continuous selections for the best approximation operator. The monograph catalogues the latest results for such settings as: nonatomic sigma finite measures, finite discrete measures, approximation to continuous functions, one-sided and two-sided approximation.

Included results vary from the elegant classical Jackson theorem that the polynomials give unique best $L_1$ approximations to continuous functions on $[0,1]$ (Lebesgue measure), to the recent result characterizing the finite-dimensional subspaces that admit best $L_1$ approximations for all reasonable “admissible” measures. The last chapter develops a gradient method algorithm for computing approximations in each setting.

The book is a scholarly presentation with attention given to technical details. Other worthy features of the book include: problems after each chapter outlining digressive topics, thorough attention to historical references, and appendices on WT-systems and convexity cones. The book could easily be used in a graduate topics course, and surely will be a standard reference work.  

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