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The exact asymptotic value for the $N$-width of smooth functions in $L^\infty$.


The authors compute the exact asymptotic value for the $N$-width of the set $D_r = \{ f : \|Lf\|_{\infty} \leq 1, f \in W_r^\infty[0,1] \}$ where $L$ is an $r$-th order totally disconjugate differential operator. This result answers a question of C. K. Chui and the reviewer. In particular it is shown that $N^r$ times the $N$-width of $D_r$ is

$$e_r = \frac{2}{\pi^{r+1}} \sum_{-\infty}^{\infty} (-1)^k (r+1)/(2k+1)^{r+1}$$

which is the sup norm of the Euler polynomial of degree $r$ on $[0,1]$.

The proof draws on results in total positivity.

{For the entire collection see MR 54 #5673.} Philip W. Smith