On positive harmonic functions in cones and cylinders

Alano Ancona
Université de Paris-Sud

Starting from a question raised by Alexander Eremenko about a uniqueness property for positive harmonic functions in an arbitrary (connected) open cone in $\mathbb{R}^d$, we investigate the Martin boundary at infinity of such cones. Some related results connected with the dimension of the cone will also be described and examples will be given showing they are sharp. Some other results connected with a regularity assumption will also be described. Most or the results under discussion extend to cylinders $\Sigma \times \mathbb{R}$ equipped with a suitable elliptic operator, where Σ is a relatively compact domain in a manifold.

Large solutions to semilinear elliptic equations with Hardy potential and exponential nonlinearity

Catherine Bandle
University of Basel

On a bounded smooth domain $\Omega \subset \mathbb{R}^N$ we study solutions of a semilinear elliptic equation with an exponential nonlinearity and a Hardy potential depending on the distance to $\partial\Omega$. We derive global a priori bounds of the Keller–Osserman type. Using a Phragmen–Lindelöf alternative for generalized sub and super-harmonic functions we discuss existence, nonexistence and uniqueness of so-called *large* solutions, i.e., solutions which tend to infinity at $\partial\Omega$. The approach develops the one used by Bandle, Moroz, Reichel for a problem with a power nonlinearity instead of the exponential nonlinearity.

Finite extinction time for non-negative solutions of some parabolic equations

Yves Belaud
Université Francois Rabelais, Tours

We consider Ω a bounded regular domain and the following equation :

$$\begin{cases} u_t + Lu + a(x)\, u^q = 0 \text{ on } \Omega, \\ a \geq 0 \ \text{ a.e.}, \quad a \in L^\infty(\Omega), \ 0 < q < 1. \end{cases}$$

for the Dirichlet or Neumann boundary condition where L is a suitable operator. Here, the function a is degenerate, i.e., it vanishes on sets of positive measure.
We try to answer the following question : is there a finite time T such that $u(T) = 0$?
We will present all the results coming from so-called semi-classical methods. The first semi-classical method was initiated in 1997 by V.A. Kondratiev and L. Véron, improved in 2001 by Y.B., B. Helffer and L. Véron and optimized with A. Shishkov in 2007 for $Lu = -\Delta u$.
With A. Shishkov, we initiate a new semi-classical method which allow us to tackle high order operators, that is, $Lu = (-1)^m \Delta(\Delta(...(\Delta u)...))$ m times.
In 2001, some results have been established when $Lu = -\Delta_p u$, $Lu = -\Delta(u^m)$. Now, with A. Shishkov, our aim is to improve them.

Scalar conservation laws on a half-line: A parabolic approach

Matania Ben-Artzi
Institute of Mathematics, Hebrew University of Jerusalem

The initial-boundary value problem for the (viscous) nonlinear scalar conservation law is considered:

$$\begin{aligned} (u_\varepsilon)_t + f(u_\varepsilon)_x &= \varepsilon (u_\varepsilon)_{xx}, \quad x \in \mathbb{R}_+ := (0,\infty)\ , \quad 0 \leq t \leq T, \quad \varepsilon > 0, \\ u_\varepsilon(x,0) &= u_0(x), \\ u_\varepsilon(0,t) &= g(t). \end{aligned}$$

The flux $f(\xi) \in C^2(\mathbb{R})$ is assumed to be convex (but not strictly convex, i.e. $f''(\xi) \geq 0$).

It is shown that a unique limit $u = \lim\limits_{\varepsilon \to 0} u_\varepsilon$ exists .

The classical duality method is used to prove uniqueness. To this end, parabolic estimates for both the direct and dual solutions are obtained. In particular, no use is made of the Kružkov entropy considerations.

(joint work with Miriam Bank)

Dirichlet problems with singular convection terms and applications

Lucio Boccardo
University of Rome

In a recent paper, dedicated to the memory of Guido Stampacchia in the thirtieth anniversary of his death, I improved some of his results concerning the Dirichlet problem

$$\begin{cases} -\mathrm{div}(M(x)\nabla u) + u = -\mathrm{div}(u\,E(x)) + f(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded, open subset of $\mathbb{R}^N$, $N > 2$, $M : \Omega \times \mathbb{R} \to \mathbb{R}^{N^2}$, is a bounded and measurable matrix such that

$$\alpha|\xi|^2 \leq M(x)\xi\xi, \quad |M(x)| \leq \beta,$$

and

$$f \in L^m(\Omega),\ \ 1 \leq m,\ E \in (L^N(\Omega))^N.$$

Now, we consider equations with vector fields E which do not belong to $(L^N(\Omega))^N$. Note that we do not assume $\mathrm{div}(E) = 0$. The most important aim of the talk is the study of the case $E \in (L^2(\Omega))^N$, where one of the main points is the definition of solution.

Then the techniques used to study problem (1) are used for the study of an elliptic system of two equations .

Quasistatic evolution for Cam-Clay plasticity

Gianni Dal Maso
Università di Roma I, Rome

Cam-Clay nonassociative plasticity exhibits both hardening and softening behavior, depending on the loading. For many initial data the classical formulation of the quasistatic evolution problem has no smooth solution. We propose here a notion of generalized solution, based on a viscoplastic approximation. To study the limit of the viscoplastic evolutions we rescale time, in such a way that the plastic strain is uniformly Lipschitz with respect to the rescaled time. The limit of these rescaled solutions, as the viscosity parameter tends to zero, is characterized through an energy- dissipation balance, that can be written in a natural way using the rescaled time. We can also give a differential characterization of the limit, based on a generalization of the flow rule in the rescaled time. It turns out that the proposed solution may be discontinuous with respect to the original time, and our formulation allows to compute the amount of viscous dissipation occurring instantaneously at each discontinuity time.

This is a joint work with Antonio DeSimone and Francesco Solombrino.

On the differentiability of very weak solutions with right hand side data integrable with respect to the distance to the boundary

J.I. Díaz
Universidad Complutense de Madrid

One of the starting arguments in the series of papers by Laurent Véron and coauthors, also used by Marie-Francoise Bidaut-Véron in some of her papers, is the solvability of the Dirichlet problem associated to second order semilinear elliptic equations $Lu + \beta(u) = f$ in terms of the notion of very weak solution.

That this notion can be applied to the case in which the right hand side f is merely in $L^1(\Omega, \delta)$, with $\delta = \text{dist}\,(x, \partial\Omega))$, was originally proved by Haim Brezis, at the late seventies, in a famous unpublished manuscript (see also his paper, published in 1996, in collaboration with T. Cazenave, Y. Martel, and A. Ramiandrisoa). But the question of the global differentiability properties of those very weak solutions, already raised by Brezis in his unpublished manuscript, remained open since then, even for the case of very weak solutions of $Lu = f$.

In this talk, I will present a series of results, obtained in collaboration with Jean-Michel Rakotoson (Poitiers, France), about such a question. Among other tools, we use symmetrization techniques to derive the searched regularity in suitable Lorentz spaces. The application to semilinear equations of the type $Lu - Vu + g(x, u, \nabla u) = \mu$ will be also presented.

Homoclinic and heteroclinic orbits for a semilinear parabolic equation

Marek Fila
Comenius University, Bratislava

We study the existence of connecting orbits for the Fujita equation

$$u_t = \Delta u + u^p$$

with a critical or supercritical exponent p. For certain ranges of the exponent we prove the existence of heteroclinic connections from positive steady states to zero and the existence of a homoclinic orbit with respect to zero.

Higher order elliptic problems in non-smooth domains

V.Maz'ya,
University of Liverpool and University of Linkoeping

We discuss sharp continuity and regularity results for solutions of the polyharmonic equation in an arbitrary open set. The absence of information about geometry of the domain puts the question of regularity properties beyond the scope of applicability of the methods devised previously, which typically rely on specific geometric assumptions. Positive results have been available only when the domain is sufficiently smooth, Lipschitz or diffeomorphic to a polyhedron.

The techniques developed recently allow to establish the boundedness of derivatives of solutions to the Dirichlet problem for the polyharmonic equation under no restrictions on the underlying domain and to show that the order of the derivatives is maximal. An appropriate notion of polyharmonic capacity is introduced which allows one to describe the precise correlation between the smoothness of solutions and the geometry of the domain.

We also study the 3D Lamé system and establish its weighted positive definiteness for a certain range of elastic constants. By modifying the general theory developed by Maz'ya (Duke, 2002), we then show, under the assumption of weighted positive definiteness, that the divergence of the classical Wiener integral for a boundary point guarantees the continuity of solutions to the Lamé system at this point.

The lectures are based on my work and recent joint papers with S. Mayboroda (Purdue) and Guo Luo (Caltech).

Singularities and phases of circle-valued maps

Petru Mironescu
Université Lyon 1

We give the description of the space

$$X := \{u : \Omega \to \mathbb{S}^1;\ u \in W^{s,p}\}.$$

Here, $0 < s < \infty$, $1 \leq p < \infty$ and Ω is a smooth bounded domain or manifold.

Maps in X can be characterized with the help of two objects: phases and topological singularities.

We will show why phases may lose or gain regularity with respect to the regularity of the initial map u.

We will explain what are the topological singularities and how to construct maps with prescribed singularities.

The final result, which identifies maps with a couple (phase, topological singularities), is a Sobolev spaces version analog of Weierstrass' factorization theorem for holomorphic maps, which allows to identify holomorphic maps with a couple (logarithm, zeroes).

A priori estimate and existence for elliptic problems with subquadratic gradient dependent terms

François Murat
Université Pierre et Marie Curie (Paris VI)

In this lecture I will consider the nonlinear elliptic model problem

$$u \in H_0^1(\Omega), \qquad -\mathrm{div}\, A(x)Du + \alpha_0 u = \gamma |Du|^q + f(x) \ \text{ in } \mathcal{D}'(\Omega),$$

with A a coercive matrix with bounded coefficients, $\alpha_0 \geq 0$, $0 \leq q \leq 2$ and $f \in L^m(\Omega)$ for some suitable m. This is a model problem, and there are many possible variants of it. In particular, I will consider the case where the right-hand side is only bounded by (but not equal to) $\gamma|Du|^q + f(x)$.

In the case where $0 \leq q < 1$, existence is classical for $f \in H^{-1}(\Omega)$. When γ is large, the case where $q = 1$ and $f \in H^{-1}(\Omega)$ is difficult but has been solved by G. Bottaro and M.E. Marina in 1973. On the other hand, the case where $q = 2$ has been treated by many authors, and in particular in a series of papers by L. Boccardo, J.-P. Puel and myself, where we showed the existence of a solution $u \in H_0^1(\Omega) \cap L^\infty(\Omega)$ (and an a priori estimate in these spaces) when $\alpha_0 > 0$ and $f \in L^m(\Omega)$ with $m > N/2$. Then V. Ferone and myself proved the existence of a solution $u \in H_0^1(\Omega)$ with $\exp(\gamma u) - 1 \in H_0^1(\Omega)$ (and an a priori estimate in these spaces) when $\alpha_0 = 0$ and $f \in L^{N/2}(\Omega)$ is sufficiently small in $L^{N/2}(\Omega)$. The case $\alpha_0 > 0$ and f arbitrary in $L^{N/2}(\Omega)$ has been treated by A. Dall'Aglio, D. Giachetti and J.-P. Puel.

In this lecture I will mainly report about a joint work with N. Grenon and A. Porretta, the announcement of which has been published in C. R. Acad. Sci. Paris, Série I 342 (2006), 23-28. When $1+(2/N) \leq q < 2$ and $f \in L^m(\Omega)$ with $m = N(q-1)/q$ (we also solved the case where $1 \leq q < 1+(2/N)$, but I will not discuss it since it uses the notion of renormalized solution), and when either $\alpha_0 > 0$ or f is sufficiently small in $L^m(\Omega)$, we prove the existence of a solution $u \in H_0^1(\Omega)$ with $|u|^\sigma \in H_0^1(\Omega)$ with $\sigma = (N-2)(q-1)/2(2-q)$. (Uniqueness results in this class have also been proved by G. Barles and A. Porretta under certain structure conditions.) Even more important that this existence result, we prove an a priori estimate for every solution of the problem which belongs to this class. The proof of this a priori estimate is non standard.

Remarks on elliptic problems involving the Hardy potential

Ireneo Peral
Universidad Autonoma de Madrid

We study the semi-linear elliptic problems involving the Hardy potential

$$-\Delta u = f\Big(\frac{\lambda}{|x|^2}, u\Big)$$

in a bounded domain Ω and with convenient hypotheses on f.

We will focus the attention on the role that plays the position of the pole in $\overline{\Omega}$.

This is a joint work with J. D. Dávila from Universidad de Chile, Santiago (Chile).

The Fujita exponent for semilinear heat equations with quadratically decaying potential or in an exterior domain

Ross Pinsky
Technion-Israel Institute of Technology

Consider the equation

$$\begin{aligned} &u_t = \Delta u - Vu + au^p \ \text{ in } \mathbb{R}^n \times (0,T), \\ &u(x,0) = \phi(x) \gneq 0 \ \text{ in } \mathbb{R}^n, \end{aligned} \tag{2}$$

where $p > 1$, $n \geq 2$, $T \in (0,\infty]$, $V(x) \sim \frac{\omega}{|x|^2}$ as $|x| \to \infty$, for some $\omega \neq 0$, and $a(x)$ is on the order $|x|^m$ as $|x| \to \infty$, for some $m \in (-\infty,\infty)$. A solution to the above equation is called *global* if $T = \infty$. Under some additional technical conditions, we calculate a critical exponent $p^* = p^*(\omega, m)$ such that global solutions exist for $p > p^*$, while for $1 < p \leq p^*$, all solutions blow up in finite time. There is a discontinuity in p^* at $\omega = -\frac{1}{4}(n-2)^2$, which is due to certain spectral theoretic considerations concerning the corresponding linear problem. We also show that when $V \equiv 0$, the blow-up/global solution dichotomy for (2) coincides with that for the corresponding problem in an exterior domain with the Dirichlet boundary condition, including the case in which p is equal to the critical exponent.

Critical nonlinearities in partial differential equations

Stanislav Pohozaev
Steklov Mathematical Institute

The talk is devoted to the blow-up theory of nonlinear equations. The main content of the talk is representation and demonstration of a new approach to the blow-up problems. This approach is based on the notion of nonlinear capacity generated by a nonlinear operator. By this way we reduce the blow-up problem to the variational one for the equations under consideration, including nonlinear elliptic, parabolic and hyperbolic equations of the second and higher orders. This approach is demonstrated on a number of concrete examples of nonlinear elliptic, parabolic and hyperbolic equations.

How do isolated singularities look like?

Augusto Ponce
Université catholique de Louvain

In the talk we will explain how a positive solution of the equation

$$-\Delta u = u^q$$

behaves when we approach one of its isolated singularities. Some important tools in this problem are:

- an a priori bound of the solutions;
- uniqueness of solutions of an elliptic PDE on the sphere.

These issues will be discussed following our recent work with M.-F. Bidaut-Véron and L. Véron.

On the construction of p-harmonic functions in a cone

Alessio Porretta
University of Rome "Tor Vergata"

If (r, s) denote spherical coordinates in N dimensions, we discuss the existence of functions $u = r^b g(s)$ which are p-harmonic in the cone C_S with vertex in the origin and opening S, where S is a subset of the N-dimensional unit sphere. In '83 P. Tolksdorf proved that, for any subdomain S of the sphere, there is a unique positive exponent b and a unique (up to a multiplicative constant) function $g(s)$ such that u is p-harmonic. In a joint work with L. Véron, we give a totally different proof of Tolksdorf 's result based on a new construction of the couple (b, g) in terms of intrinsic quasilinear problems on the sphere.

Singularities versus boundedness of solutions of superlinear elliptic boundary value problems

Pavol Quittner
Comenius University, Bratislava

We study the impact of boundary conditions on the existence of singularities of very weak solutions of superlinear elliptic boundary value problems.

After presenting recent results on scalar problems, we will consider the model elliptic system

$$-\Delta u = f(x, v), \quad -\Delta v + v = g(x, u)$$

in a bounded smooth domain Ω. Here f, g are nonnegative Carathéodory functions satisfying the growth conditions

$$f \leq C(1 + |v|^p), \quad g \leq C(1 + |u|^q),$$

and we complement the system with one of the following three boundary conditions on $\partial\Omega$:

$$\begin{aligned} &u = v = 0 \text{ (Dirichlet)},\\ &\partial_\nu u = \partial_\nu v = 0 \text{ (Neumann)}\\ &u = \partial_\nu v = 0 \text{ (Dirichlet-Neumann)}. \end{aligned}$$

In all three cases we find optimal conditions on p, q guaranteeing that all nonnegative solutions belong to $L^\infty(\Omega)$ (and satisfy suitable a priori estimates).

We also consider related problems in non-smooth domains and problems with nonlinear boundary conditions.

Inverse spectral results for Schrödinger operators on the unit interval

Thierry Raoux
Université de Reims

The results presented here, obtained with L. Amour and J. Faupin, are concerned with the inverse spectral analysis of the Schrödinger operator $A_{q,h,H} = -\frac{d^2}{dx^2} + q$ defined on $[0,1]$ associated with the boundary conditions $u'(0) + hu(0) = u'(1) + Hu(1) = 0$ (where $q \in L^1((0,1),\mathbb{R})$ and $h, H \in \mathbb{R}$).

The starting point of our study is a paper of F. Gesztesy and B. Simon, published in 1999, which established the following theorem. Suppose that the potentials q_1 and q_2 are equal on some interval $[a,1]$ and that the spectra of $A_{q_1,h,H}$, $A_{q_2,h,H}$ have a common part, which is sufficiently large in terms on a, for some pair $(h,H) \in \mathbb{R}^2$. Then $q_1 = q_2$ on $(0,1)$. Moreover, the condition on the common part of the spectra can be weakened if q_1, q_2 are C^k in some neighborhood of a. This largely extends a theorem proved in 1978 by H. Hochstadt and B. Lieberman

In our results, whose proofs are based upon a different method, the potentials are supposed to belong to L^p or $W^{k,p}$ spaces and we give other sufficient conditions on the size of the common part of the spectra. In certain cases, this allows us to improve the results of the previous authors. For example, we obtain that one of their conclusions remains true, with q_1, q_2 in L^∞ instead of C^0, which could be useful for applications. Moreover, we are able to prove that the values of q on $[\frac{1}{4}, 1]$ and the odd (resp. even) part of the spectrum of $A_{q,h,H}$ determine q on $(0,1)$.

Entire solutions of completely coercive quasilinear elliptic equations

James Serrin
University of Minnesota

We prove the following generalization of a theorem of H. Brezis. Consider the equation

$$(*) \qquad \Delta_{p,r}\, u = b(x, u, Du), \qquad x \in \mathbf{R}^n,$$

where

$$\Delta_{p,r}\, u = \operatorname{div}\{|u|^r\, |Du|^{p-2}\, Du\}, \qquad p > 1, \quad r \geq 0.$$

Suppose that

$$b(x, u, Du)\,\operatorname{sign} u \geq \begin{cases} 0 & \text{if } |x| \leq R_0, \\ f(u)\,|Du|^\ell & \text{if } |x| > R_0, \end{cases}$$

where

$$\begin{cases} f(u) \geq 0 \quad \text{for } |u| > 0; \quad p > n, \\ f(u) \text{ is positive and continuous for } |u| > 0; \quad p \leq n, \end{cases}$$

and

$$f(u) = |u|^q \quad \text{when } |u| \geq d, \qquad d = \text{const.} \geq 0.$$

Let

$$q + \ell \geq p + r - 1, \qquad \ell \leq p - 1.$$

If $u = u(x)$ is a C^1 entire solution of $(*)$, then u must be identically constant in $\mathbf{R}^n$.

Brezis (1984) proved the special case

$$p = 2, \quad d = \ell = r = R_0 = 0,$$

using different methods which do not carry over to the present case.

Very singular solutions of semi-linear parabolic equations with degenerate absorption potential

Andrey Shishkov
NAS of Ukraine, Donetsk

We study the limit behaviour as $k \to \infty$ of solutions $u_k(t,x)$ of the Cauchy problem:

$$u_t - \Delta u + h(t,x)u^p = 0 \quad \text{in} \quad \mathbb{R}^N \times (0,\infty), \quad u(0,x) = k\delta_0(x),\ p > 1,$$

with nonnegative continuous potential h such that

$$h(t,x) = 0\ \ \forall\,(t,x) \in \Gamma,\ \ (0,0) \in \Gamma,$$

where Γ is some smooth manifold in $\mathbb{R}_+^{N,1} := \{x \in \mathbb{R}^N,\ t \geq 0\}$. Particularly, if $h(t,x) = h(|x|) = |x|^\beta,\ \beta > 0$ ($\Gamma = \{(t,0) : t \geq 0\}$), then the limit function $u_\infty(t,x)$ is an explicit very singular solution (solution with infinite initial mass) in the case $\beta > N(p-1)-2$, while such a very singular solution does not exist if $\beta \leq N(p-1)-2$.

Essentially new phenomenon happens if the potential $h(|x|)$ is very flat near to Γ.
a) If $\lim_{r\to 0}\inf r^2 \ln(\frac{1}{h(r)}) > 0$ ($h(|x|) \sim \exp(-c|x|^{-2})$), then $u_\infty(t,x)$ has a persistent singularity at arbitrary $(t,0) \in \Gamma$ ("razor blade solution").

Otherwise, if
b) $\int_0^1 r \ln(\frac{1}{h(r)})\, dr < \infty$ (Dini-like condition), then $u_\infty(t,x)$ is the usual very singular solution with pointwise singularity at $(0,0)$.

We study the mentioned phenomenon of propagation-nonpropagation of singularities of solutions for a wide class of quasilinear parabolic diffusion-absorption type equations with different classes of degenerate potentials $h(t,x)$. Nonpropagation property is treated by a new variant of local energy estimate method, main steps of which we are going to describe in the talk.

Joint work with Laurent Véron.

Asymptotic behavior of solutions of a parabolic system arising in a selection-migration model in population genetics

Philippe Souplet
University of Paris 13

We study a mathematical model from population genetics, describing a single-locus diallelic (A/a) selection-migration process. The model consists of a coupled system of three reaction-diffusion equations, one for the density of each genotype, posed in the whole space $\mathbb{R}^n$. The genotype AA is advantageous, due to a smaller death rate, and we consider the fully recessive case where the other two genotypes aa and Aa have the same (higher) death rate. In the nondiffusive (spatially homogeneous) case, the disadvantageous gene a is always eliminated in the large time limit. In the presence of diffusion, when the birth rate exceeds a certain threshold value, we prove that this conclusion is still true for dimensions $n \leq 2$, whereas for $n \geq 3$ there exist initial distributions for which the advantageous gene A ultimately disappears. This is the first rigorous result of this type for the full system, and it solves a problem which seems to have been open since the celebrated work of Aronson and Weinberger (1975, 1977), where similar results had been obtained for a simplified scalar model, that they derived as an approximation of the full system. Interestingly, we moreover show that, at the threshold value of the birth rate, the cut-off dimension shifts from $n = 2$ to $n = 6$ This is joint work with Michael Winkler.

Porous medium flow with fractional diffusion

Juan Luis Vazquez
Universidad Autónoma Madrid

We study a model for flow in porous media including nonlocal (long-range) diffusion effects. It is based on Darcys law and the pressure is related to the density by an inverse fractional Laplacian operator. We prove existence of solutions that propagate with finite speed, which is unexpected in fractional diffusion models.

The model has also the very interesting property that mass preserving selfsimilar solutions can be found by solving an elliptic obstacle problem with fractional Laplacian for the pair pressure-density. We use entropy methods to show that the asymptotic behaviour is described after renormalization by these solutions which play the role of the Barenblatt profiles of the standard porous medium model.

This is a joint project with Luis Caffarelli.

Green's function estimates for some linear and nonlinear operators

Igor E. Verbitsky
University of Missouri, Columbia

Global bilateral estimates will be presented for Green's function of the fractional Schrödinger operator $Lu = (-\Delta)^{\alpha/2}u - Vu$ $(0 < \alpha \leq 2)$. Here V is a general nonnegative measurable function (or measure). These results will be deduced from sharp estimates of the kernel of the Neumann series associated with an integral operator with positive kernel on a measure space satisfying a quasimetric property.

Analogous estimates will be discussed for the p-Laplacian with a natural growth term, $-\Delta_p u - V u^{p-1}$, $1 < p < \infty$, as well as more general quasilinear and fully nonlinear elliptic operators, in particular $F_k[u] - Vu^k$ where F_k is the k-Hessian operator.

This talk is based on joint work with Michael Frazier and Fedor Nazarov, and Benjamin Jaye.

Multi-profile, multi-scale asymptotic limits of global solutions of evolution equations

Fred Weissler
University of Paris 13

We consider the (linear and nonlinear) heat equations on $\mathbb{R}^N$

$$u_t - \Delta u - \kappa |u|^\alpha u = 0, \qquad (H)$$

where $\alpha > 0$, and $\kappa = 1, -1$ or 0. Also $u = u(t,x), t > 0,\ x \in \mathbb{R}^N$.

We begin by recalling known results about (forward) self-similar solutions of (H) and asymptotically self-similar (global) solutions of (H). A solution is asymptotically self-similar if an appropriate rescaling of the solution converges as t goes to infinity to the profile of a self-similar solution.

We then discuss recent results concerning more general large time asymptotic behavior of solutions. In particular, we consider multi-profile limits. In this case, the same rescaling of a given solution has different asymptotic limits along different time sequences going to infinity. Also, the time T flow generated by (H), combined with an appropriate spatial rescaling, generates a chaotic discrete dynamical system.

In the case of the linear heat equation, it turns out as well that certain solutions admit nontrivial large time asymptotic limits with respect to multiple time scales. Even more surprisingly, there exist solutions which exhibit nontrivial large time asymptotic limits with respect to various rescalings which do not preserve the parabolic structure of (H).

Similar results hold for the linear and nonlinear Schrödinger equations, and the Navier Stokes system.

This is joint work with T. Cazenave (Université de Paris 6) and F. Dickstein (Universidade Federal do Rio de Janeiro).