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The influence of space dimension on the large-time behavior in a reaction-diffusion system modeling diallelic selection

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We study a mathematical model from population genetics, describing a single-locus diallelic (A/a) selection-migration process. The model consists in the following coupled system of three reaction-diffusion equations, one for the density ρ_i of each genotype, posed in the whole space $\mathbb{R}^n$:

$$\begin{cases} \partial_t\rho_1 = \Delta\rho_1 - \tau_1\rho_1 + \dfrac{r}{\rho}\Big(\rho_1 + \dfrac{1}{2}\rho_2\Big)^2, & x \in \mathbb{R}^n,\ t > 0, \\ \partial_t\rho_2 = \Delta\rho_2 - \tau_2\rho_2 + \dfrac{2r}{\rho}\Big(\rho_1 + \dfrac{1}{2}\rho_2\Big)\Big(\rho_3 + \dfrac{1}{2}\rho_2\Big), & x \in \mathbb{R}^n,\ t > 0, \\ \partial_t\rho_3 = \Delta\rho_3 - \tau_3\rho_3 + \dfrac{r}{\rho}\Big(\rho_3 + \dfrac{1}{2}\rho_2\Big)^2, & x \in \mathbb{R}^n,\ t > 0. \end{cases}$$

The genotype AA is advantageous, due to a smaller death rate τ_1. We mainly consider the fully recessive case where the other two genotypes aa and Aa have the same death rate $\tau_2 = \tau_3 > \tau_1$. In the nondiffusive (spatially homogeneous) case, the disadvantageous gene a is always eliminated in the large time limit. In the presence of diffusion, when the birth rate exceeds a certain threshold value, we prove that this conclusion is still true for dimensions $n \le 2$, whereas for $n \ge 3$ there exist initial distributions for which the advantageous gene A ultimately disappears. This is the first rigorous result of this type for the full system, and it solves a problem which seems to have been open since the celebrated work [1, 2] of Aronson and Weinberger (1975, 1977), where similar results had been obtained for a simplified scalar model, that they derived as an approximation of the full system. Interestingly, we moreover show that, at the threshold value of the birth rate, the cut-off dimension shifts from $n = 2$ to $n = 6$. We also study the nonrecessive cases, where we show that no dimensional effects occur.

REFERENCES

[1] D.G. Aronson, H.F. Weinberger, Nonlinear diffusion in population genetics, combustion, and nerve propagation, in "Partial Differential Equations and Related Topics, Lecture Notes in Mathematics", Vol. 446, pp. 5-49, Springer, New York, 1975.

[2] D.G. Aronson, H.F. Weinberger, Multidimensional nonlinear diffusions arising in population genetics, Adv. Math. 30 (1977), 33-76.

[3] P. Fife, Mathematical Aspects of Reacting and Diffusing Systems, Lect. Notes Biomath., vol. 28, Berlin Heidelberg New York, Springer 1979.

[4] V.A. Kostitzin, Biologie Mathématique, Librairie Armand Colin, Paris, 1937.

[5] Y. Lou, Th. Nagylaki, Evolution of a semilinear parabolic system for migration and selection without dominance, J. Differ. Equations 225 (2006), 624-665.

[6] R. Lui, Convergence to constant equilibrium for a density-dependent selection model with diffusion, J. Math. Biol. 26 (1988), 583-592.

[7] R. Lui, J.F. Selgrade, Density-dependent selection migration model with non-monotone fitness functions, J. Math. Biol. 31 (1993), 717-734.

[8] J.D. Murray, Mathematical Biology. Vol. 1: An introduction. Interdisciplinary Applied Mathematics 17, Springer, New York, 2002.

[9] Th. Nagylaki, Introduction to theoretical population genetics. Biomathematics 21, Springer, Berlin, 1992.

[10] P. Quittner, Ph. Souplet, Superlinear parabolic problems. Blow-up, global existence and steady states, Birkhäuser Advanced Texts, 2007.

[11] Ph. Souplet, M. Winkler, The influence of space dimension on the large-time behavior in a reaction-diffusion system modeling diallelic selection, J. Math Biology (2010), in press.

[12] Ph. Souplet, M. Winkler, The influence of space dimension on the large-time behavior in a reaction-diffusion system modeling diallelic selection, II: the nonrecessive case, in preparation.

[13] Yu.V. Tyutyunova, E.A. Zhadanovskayaa, R. Arditi, A.B. Medvinsky, A Spatial Model of the Development of Pest Resistance to a Transgenic Insecticidal Crop: European Corn Borer on Bt Maize, Biophysics 52 (2007), 52-67.

[14] E. Zhadonovskaya, Modelling the spatio-temporal dynamics of the European corn borer in the presence of trangenic maize, PhD Thesis, Agro-Paris-Tech, 2007.