Entire solutions of completely coercive quasilinear elliptic equations

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We prove the following generalization of a theorem of H. Brezis. Consider the equation

$$(*) \qquad \Delta_{p,r}\, u = b(x, u, Du), \qquad x \in \mathbf{R}^n,$$

where

$$\Delta_{p,r}\, u = \operatorname{div}\{|u|^r\, |Du|^{p-2}\, Du\}, \qquad p > 1, \quad r \geq 0.$$

Suppose that

$$b(x, u, Du)\operatorname{sign} u \geq \begin{cases} 0 & \text{if } |x| \leq R_0, \\ f(u)\,|Du|^\ell & \text{if } |x| > R_0, \end{cases}$$

where

$$\begin{cases} f(u) \geq 0 \quad \text{for } |u| > 0; \quad p > n, \\ f(u) \text{ is positive and continuous for } |u| > 0; \quad p \leq n, \end{cases}$$

and

$$f(u) = |u|^q \quad \text{when } |u| \geq d, \qquad d = \text{const.} \geq 0.$$

Let

$$q + \ell \geq p + r - 1, \qquad \ell \leq p - 1.$$

If $u = u(x)$ is a C^1 entire solution of $(*)$, then u must be identically constant in $\mathbf{R}^n$.

Brezis (1984) proved the special case

$$p = 2, \quad d = \ell = r = R_0 = 0,$$

using different methods which do not carry over to the present case.