Scalar conservation laws on a half-line: A parabolic approach

Matania Ben-Artzi
Institute of Mathematics, Hebrew University of Jerusalem

The initial-boundary value problem for the (viscous) nonlinear scalar conservation law is considered:

$$\begin{aligned}(u_\varepsilon)_t + f(u_\varepsilon)_x &= \varepsilon(u_\varepsilon)_{xx}, \quad x \in \mathbb{R}_+ := (0,\infty)\ , \quad 0 \le t \le T, \quad \varepsilon > 0,\\ u_\varepsilon(x,0) &= u_0(x),\\ u_\varepsilon(0,t) &= g(t).\end{aligned}$$

The flux $f(\xi) \in C^2(\mathbb{R})$ is assumed to be convex (but not strictly convex, i.e. $f''(\xi) \ge 0$).

It is shown that a unique limit $u = \lim\limits_{\varepsilon\to 0} u_\varepsilon$ exists .

The classical duality method is used to prove uniqueness. To this end, parabolic estimates for both the direct and dual solutions are obtained. In particular, no use is made of the Kružkov entropy considerations.

(joint work with Miriam Bank)